Experiment 4： The Studies of transitional processes of first-order circuit

1. Purpose

a) Use the oscilloscope to observe the transitional processes of first-order circuit.
b) Use the oscilloscope to measure the time constant of first-order circuit.
c) Know the concepts of differential circuit and integral circuit.
d) Observe the characteristics of the step response and the square-wave response of first-order circuit.

2. Principal and Illustration

A. First-order circuit and its transitional processes

The circuit containing energy-storage elements is called dynamic circuit. When the characteristics of the dynamic circuit can be described by first-order differential equation, the dynamic circuit is called first-order circuit. When the dynamic circuit is on its steady state, as its structure or parameters change, the transitional process emerges.

The transitional process of a circuit contains zero-input response, zero-state response and complete response. As shown in Figure 3-5-1(a), if the response is capacitor voltage u_c in the first-order RC circuit, its complete response can be represented by

$$u_C(t) = U_m + [u_C(0_+) - U_m]e^{-\frac{t}{\tau}} \qquad (3\text{-}5\text{-}1)$$

where $u_c(0_+)$ is the initial capacitor voltage, U_m is direct current voltage excitation, which is added into the circuit and $\tau = RC$ is the time constant.

Complete response can be regarded as the superposition of zero-input response and zero-state response.

(1) When $u_c(0_+)=0$ or the initial energy-storage capacitor is zero, we have

$$u_C(t) = U_m\left(1 - e^{-t/\tau}\right) \qquad (3\text{-}5\text{-}2)$$

This is the zero-state response, produced by the voltage excitation.

(2) When $u_s=0$, we have

$$u_C(t) = u_C(0_+)e^{-t/\tau} \qquad (3\text{-}5\text{-}3)$$

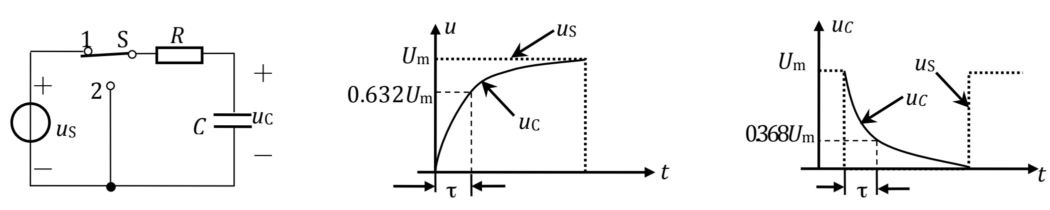
This is the zero-input response, produced by the initial energy-storage capacitor.

When the excitation is step signal, zero-state response is called step response in the circuit. The response of a first-order RC circuit to square-wave pulse sequence signal can be regarded as the superposition of several step responses.

The transitional process of dynamic circuit is one-time changing process within a very short time. For the circuit with relatively large time constant, we can use ultra-low frequency oscilloscope to observe its transitional process. However, if we use common intermediate frequency oscilloscope to observe its transitional process and measure related parameters, we need to make sure that the one-time changing process appear repeatedly. Therefore, we use the signal generator to generate square-wave signal to simulate step excitation signal in the experiment. In other words, we regarded the rising edge of the square-wave output as positive step excitation signal of zero-state response, and the falling edge of square-wave as negative step excitation signal. As long as we choose the periodic of the square-wave that is much larger than the time constant of the circuit, it can be considered that, when some edge of the principal wave is coming, the transitional process that the former edge produces has already disappeared. The transitional process excited by the square-wave sequence pulse signal is almost the same as the transitional process caused by turning on and off DC power.

B. Time constant and its measurement

When we get the waveform of response shown on the oscilloscope, we can estimate time constant τ of the circuit by the waveform. When the amplitude of capacitor voltage rises to 63.2% of final value in the zero-state response of first-order RC circuit, the corresponding time is τ, as shown in Figure 3-5-1(b). When the amplitude of capacitor voltage declines to 36.8% of starting value in the waveform of zero-input response, the corresponding time is τ, as shown in Figure 3-5-1(c).



(a) First-order RC circuit. (b) The waveform of zero-state response. (c) The waveform of zero-input response

Figure 3-5-1 RC circuit and the measurement of its time constant

C. *Differential circuit and integral circuit*

Differential circuit and integral circuit have specific requirements of the circuit element parameters and the period of input signal, and these two circuits are rather typical ones in RC one-order circuits.

If the output of first-order circuit is from the voltage across a resistance, namely $u_o = u_R$, as shown in Figure 3-5-2, where u_s is the square pulse sequence with periodic T.

When it is satisfied $\tau = RC << \frac{T}{2}$, we have

$$u_R << u_C, \quad u_C \approx u_S$$

$$u_o = u_R = RC\frac{du_C}{dt} \approx RC\frac{du_S}{dt}$$

where the output voltage u_o of the circuit is directly proportional to the differential of the input voltage u_s, and thus the circuit is called differential circuit.

When it is satisfied $\tau = RC >> \frac{T}{2}$, we have

$$u_C << u_R \quad u_S \approx u_R$$

$$u_o = u_C = \frac{1}{C}\int i_C dt = \frac{1}{C}\int \frac{u_R}{R} dt \approx \frac{1}{RC}\int u_S dt$$

where the output voltage u_o of the circuit is directly proportional to the integral of the input voltage u_s, and thus the circuit is called integral circuit.

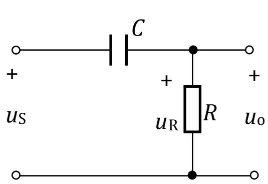


Figure 3-5-2 differential circuit.

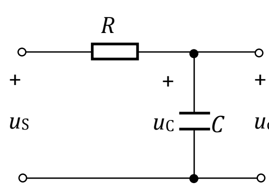


Figure 3-5-3 integral circuit.

The relationship between differential circuit and integral circuit is shown in Figure 3-5-4 and Figure 3-5-5. The waveforms of the input u_s and the corresponding output u_o are shown in Figure 3-5-4 for the differential circuit. The waveforms of the input u_s and the corresponding output u_o are shown in Figure 3-5-5 for the integral circuit.

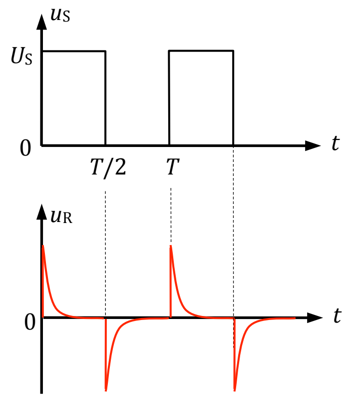


Figure 3-5-4 the waveforms of input u_s and output u_R of differential circuit

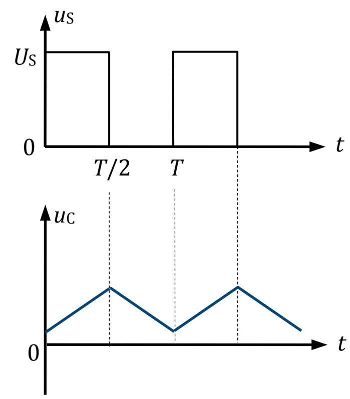


Figure 3-5-5 the waveforms of input u_s and output u_c of integral circuit

3. Contents and Steps

The experiment uses first-order circuit element board shown in Figure 3-5-6.

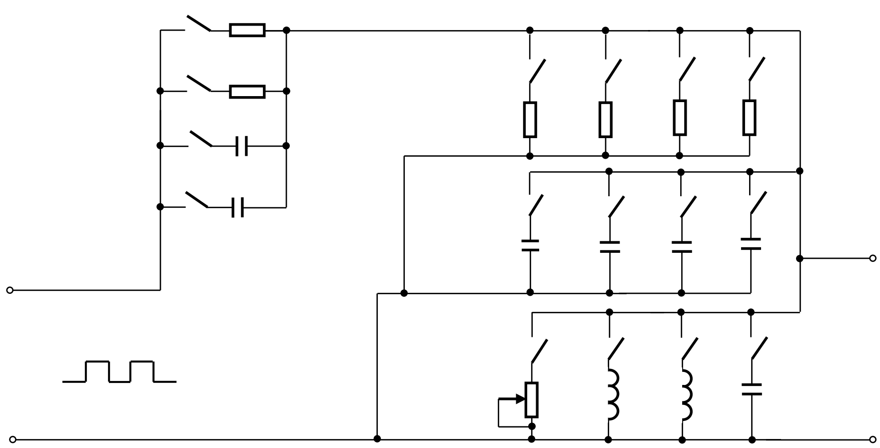


Figure 3-5-6 First-order circuit element board

A. Observe electro-charge and electro-discharge processes and the measurement of time constant in RC integral circuit

Connect the wires, as shown in Figure 3-5-6. According to the two groups of values given in Table 3-5-1, respectively, choose R and C element properly in the element board (Figure 3-5-6). The excitation u_s is square voltage signal, whose amplitude is U_m = 3V and frequency is f=1kHz. The excitation u_s and the response u_c are connected to two input channels CH1 and CH2 of the oscilloscope, respectively.

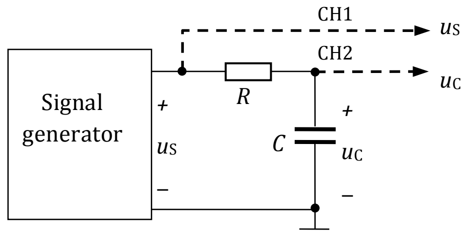


Figure 3-5-6 RC integral circuit

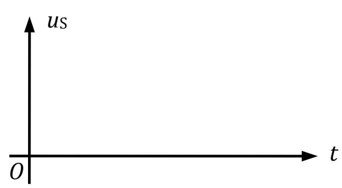
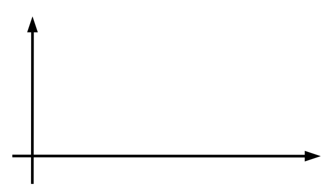
We can observe the change of excitation u_s and response u_c in the screen of oscilloscope. Complete the following experiment.

(1) Observe the constant τ from the screen of oscilloscope and record the data into Table 3-5-1.

(2) Observe the waveforms of excitation u_s and response u_c in the oscilloscope and plot the graph proportionally in the table respectively (please use graphic paper, bought from the shop, to plot the graph and the proportion is 1:1).

(3) Steadily increase the value of C, observe its influence on u_c qualitatively and record the data.

Table 3-5-1 The electro-charge and electro-discharge processes of different parameters in the RC integral circuit.

Parameters		R = 10kΩ，C = 3300pf	R = 10kΩ，C = 0.01μf
Time constant τ (μs)	Theoretical value		
	Observed value		
Observed waveforms			
			

B. Observe the waveforms of RC differential circuit

Connect the wires as shown in Figure 3-5-7. According to the two groups of values given in Table 3-5-3, respectively, choose R and C element properly in the element board (Figure 3-5-5). The excitation u_S is square voltage signal, whose amplitude is U_m = 3V and frequency is f=1 kHz.

(1) Observe the waveforms of excitation u_s and response u_R in the oscilloscope and plot the waveforms of response u_R in the Table 3-5-2 respectively (please use graphic paper, bought from the shop, to plot the graph and the proportion is 1:1).

(2) Steadily increase the value of R, observe its influence on the response qualitatively and record data.

(3) When R increases to 1 MΩ, observe the essential difference between input and output waveforms and record data.

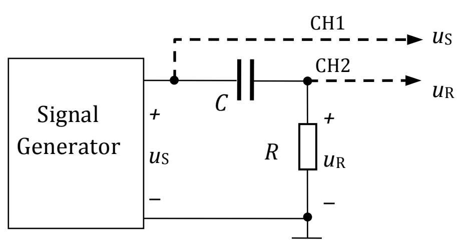


Figure 3-5-7 RC differential circuit

Table 3-5-2 The electro-charge and electro-discharge processes of different parameters in the RC differential circuit.

Parameters	C = 0.01μf，R = 1kΩ	C = 0.01μf，R = 10kΩ
Observed waveforms		

4. Questions

(1) What electrical signals can be regarded as the excitation signal of zero-input response, zero-state response and complete response in the RC one-order circuit?

(2) Suppose R=10kΩ and C=0.1μF, try to calculate the time constant τ, and according to physical meaning of τ, try to figure out how to measure τ practically.

(3) What are integral circuit and differential circuit? What conditions should they possess? Given the excitation of square wave sequence pulse, how do their output signal waveforms change?

(4) In the RC circuit, when R or C varies, what's the influence on the response of the circuit?

5. Writing Your Report

(1) Complete the calculation of Table 3-5-1.

(2) According to your experimental result, plot the curves of electric-charge and electric-discharge, differential circuit and integral circuit of RC first-order circuit in your graphic paper.

(3) According to the curve of electric-charge and electric-discharge of RC first-order circuit, estimate the time constant τ.

(4) Answer the questions in the 4^{th} section.