Problem # 1

Find the average power delivered by the ideal current source in the circuit if \( i_g = 4 \cos 5000t \text{ mA} \).

Solution:

\[ I_g = 4/0^\circ \text{ mA}; \quad \frac{1}{j\omega C} = -j1250 \Omega; \quad j\omega L = j500 \Omega \]

\[ Z_{eq} = 500 + [-j1250 || (1000 + j500)] = 1500 - j500 \Omega \]

\[ P_g = -\frac{1}{2} |I|^2 \text{Re}\{Z_{eq}\} = -\frac{1}{2} (0.004)^2 (1500) = -12 \text{ mW} \]

The source delivers 12 mW of power to the circuit.
Problem #2
Find the rms value of the periodic current.

Solution:

\[ i(t) = 250t \quad 0 \leq t \leq 80 \text{ ms} \]

\[ i(t) = 100 - 1000t \quad 80 \text{ ms} \leq t \leq 100 \text{ ms} \]

\[ I_{\text{rms}} = \sqrt{\frac{1}{0.1} \left\{ \int_0^{0.08} (250)^2 t^2 \, dt + \int_{0.08}^{0.1} (100 - 1000t)^2 \, dt \right\}} \]

\[ \int_0^{0.08} (250)^2 t^2 \, dt = (250)^2 \frac{t^3}{3} \bigg|_0^{0.08} = \frac{32}{3} \]

\[ (100 - 1000t)^2 = 10^4 - 2 \times 10^5 t + 10^6 t^2 \]

\[ \int_{0.08}^{0.1} 10^4 \, dt = 200 \]

\[ \int_{0.08}^{0.1} 2 \times 10^5 t \, dt = 10^5 t^2 \bigg|_{0.08}^{0.1} = 360 \]

\[ 10^6 \int_{0.08}^{0.1} t^2 \, dt = \frac{10^6}{3} t^3 \bigg|_{0.08}^{0.1} = \frac{488}{3} \]

\[ I_{\text{rms}} = \sqrt{10\left\{\frac{32}{3} + 225 - 360 + \frac{488}{3}\right\}} = 11.55 \text{ A} \]
Problem #3

The voltage $V_g$ shown in the frequency-domain circuit is $240\angle 0^\circ$ V (rms).

a) Find the average and reactive power for the voltage source.

b) Is the voltage source absorbing or delivering average power?

c) Is the voltage source absorbing or delivering magnetizing vars?

d) Find the average and reactive powers associated with each impedance branch in the circuit.

e) Check the balance between delivered and absorbed average power.

f) Check the balance between delivered and absorbed magnetizing vars.

Solution:

\[
\begin{align*}
\frac{V_o}{-j25} + \frac{V_o - 240}{12.5} + \frac{V_o}{15 + j20} &= 0 \\
\therefore V_o &= 183.53 - j14.12 = 184.07/ -4.4^\circ \text{V} \\
I_g &= \frac{240 - 183.53 + j14.12}{12.50} = 4.52 + j1.13 \text{ A} \\
S_g &= -V_gI_g^* = -(240)(4.52 - j1.13) \\
&= -1084.24 + j271.06 \text{ VA}
\end{align*}
\]
Problem #4
The three loads in the circuit can be described as follows: Load 1 is a 240 \( \Omega \) resistor in series with an inductive reactance of 70 \( \Omega \); load 2 is a capacitive reactance of 120 \( \Omega \) in series with a 160 \( \Omega \) resistor; and load 3 is a 30 \( \Omega \) resistor in series with a capacitive reactance of 40 \( \Omega \). The frequency of the voltage source is 60 Hz.

a) Give the power factor and reactive factor of each load.

b) Give the power factor and reactive factor of the composite load seen by the voltage source.
Solution:

[a] \[ Z_1 = 240 + j70 = 250/16.26^\circ \Omega \]

\[ \text{pf} = \cos(16.26^\circ) = 0.96 \text{ lagging} \]
\[ \text{rf} = \sin(16.26^\circ) = 0.28 \]

\[ Z_2 = 160 - j120 = 200/-36.87^\circ \Omega \]

\[ \text{pf} = \cos(-36.87^\circ) = 0.8 \text{ leading} \]
\[ \text{rf} = \sin(-36.87^\circ) = -0.6 \]

\[ Z_3 = 30 - j40 = 50/-53.13^\circ \Omega \]

\[ \text{pf} = \cos(-53.13^\circ) = 0.6 \text{ leading} \]
\[ \text{rf} = \sin(-53.13^\circ) = -0.8 \]

[b] \[ Y = Y_1 + Y_2 + Y_3 \]

\[ Y_1 = \frac{1}{250/16.26^\circ}; \quad Y_2 = \frac{1}{200/-36.87^\circ}; \quad Y_3 = \frac{1}{50/-53.13^\circ} \]

\[ Y = 19.84 + j17.88 \text{ mS} \]

\[ Z = \frac{1}{Y} = 37.44/-42.03^\circ \Omega \]

\[ \text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading} \]
\[ \text{rf} = \sin(-42.03^\circ) = -0.67 \]
Problem #5

Prove that if only the magnitude of the load impedance can be varied, most average power is transferred to the load when $|Z_L| = |Z_{Th}|$. (Hint: In deriving the expression for the average load power, write the load impedance ($Z_L$) in the form $Z_L = |Z_L|\cos \theta + j|Z_L|\sin \theta$, and note that only $|Z_L|$ is variable.)

Solution:

$$Z_L = \frac{|Z_L|e^{j\theta}}{|Z_L| = |Z_L|\cos \theta + j|Z_L|\sin \theta}$$

Thus

$$|I| = \frac{|V_{Th}|}{\sqrt{(R_{Th} + |Z_L|\cos \theta)^2 + (X_{Th} + |Z_L|\sin \theta)^2}}$$

Therefore

$$P = \frac{0.5|V_{Th}|^2|Z_L|\cos \theta}{(R_{Th} + |Z_L|\cos \theta)^2 + (X_{Th} + |Z_L|\sin \theta)^2}$$

Let $D = \text{denominator in the expression for } P$, then

$$\frac{dP}{d|Z_L|} = \frac{(0.5|V_{Th}|^2 \cos \theta)(D \cdot 1 - |Z_L|dD/d|Z_L|)}{D^2}$$

$$\frac{dD}{d|Z_L|} = 2(R_{Th} + |Z_L|\cos \theta) \cos \theta + 2(X_{Th} + |Z_L|\sin \theta) \sin \theta$$

$$\frac{dP}{d|Z_L|} = 0 \text{ when } D = |Z_L| \left( \frac{dD}{d|Z_L|} \right)$$

Substituting the expressions for $D$ and $(dD/d|Z_L|)$ into this equation gives us the relationship $R_{Th}^2 + X_{Th}^2 = |Z_L|^2$ or $|Z_{Th}| = |Z_L|$. 