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Consensus for second-order agent dynamics with velocity estimators via pinning control

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Abstract: A consensus problem is investigated for a group of second-order agents with an active leader where the velocity of the leader cannot be measured, and the leader as well as all the agents is governed by the same non-linear intrinsic dynamics. To achieve consensus in the sense of both position and velocity, a neighbour-based estimator design approach and a pinning-controlled algorithm are proposed for each autonomous agent. It is found that all the agents in the group can follow the leader, and the velocity tracking errors of estimators converge to zero asymptotically, without assuming that the interaction topology is strongly connected or contains a directed spanning tree. In terms of the switching topologies between the leader and the followers, similar results are obtained. Finally, these theoretical results are demonstrated by the numerical simulations.

1 Introduction

Over the past few years, the consensus problem of multi-agent systems has attracted much attention from the researchers of different areas, including system control theory, statistical physics, biology, applied mathematics and computer science [1–5]. Generally, the main objective of consensus problem is to design an appropriate algorithm or interaction rule such that a group of agents converges to a consistent quantity of interest. Research on the consensus problem of multi-agent systems not only helps engineers better understand the mechanisms of natural collective phenomena, such as avoiding predators and increasing the chance of finding food, but also provides useful ideas for developing formation control, distributed cooperative control and coordination of multiple mobile autonomous agents/robots [6–8].

The individuals in many practical systems, especially in mechanical systems, usually have second-order dynamics, where the aim is to guide all the agents to move with the same velocity and converge to the same position. For example, Yu *et al.* [9] studied some necessary and sufficient conditions for second-order consensus in directed networks containing a directed spanning tree. It was proved that both the real and imaginary parts of the eigenvalues of the Laplacian matrix of the communication graph play key roles in reaching consensus [9]. Qin *et al.* [10] investigated two kinds of second-order consensus strategies for multi-vehicle systems with a time-varying reference velocity under directed communication topology, and it was shown that all the vehicles can reach consensus even though the dynamically changing interaction topology may not have a globally reachable node. Furthermore, Xie and Wang [11] investigated the consensus problem of second-order agents with switching topologies and established a linear consensus protocol for solving such a consensus problem. Recently, Zheng *et al.* [12] considered the consensus problem of heterogeneous multi-agent system composed of first-order and second-order agents, for which the consensus protocols have both position and velocity information. Wen *et al.* [13] studied the second-order consensus problem with communication constraints, where each agent is assumed to share information only with its neighbours at some disconnected time intervals, which is then solved by a novel protocol with synchronous intermittent information feedback.

In the nature, the agents always have certain purposes such as achieving the desired common speed and heading, or arriving at a desired destination, which give rise to an interesting and important topic of leader–follower approach. In this approach, either an agent is chosen or a virtual one is designed as the leader, whose movement is constrained by a predefined trajectory. Then, the remaining agents track the leader according to some appropriate controllers [14–17]. In order to reduce the number of controllers, a natural approach is to control a multi-agent system by pinning a small fraction of agents, known as the pinning control scheme [18–22].

For instance, Grigoriev *et al.* [18] studied the pinning control of spatiotemporal chaos, and then Parekh *et al.* [19] investigated the global and local control of spatiotemporal chaos in coupled map lattices. Moreover, Wang and Chen [20] applied the pinning control to a scale-free dynamical network, where it was shown that the number of controllers can be significantly reduced by pinning the nodes with large degree. Yu *et al.* [21] studied some technique problems of the pinning control scheme, and found that the nodes with low degrees should be pinned first when the coupling strength is small. Recently, Qin *et al.* [22] investigated the pinning control of complex networks under arbitrary topological structures with a special focus on the case with directed graph topology, where it was found that the pinning control of complex networks relies totally on the way to pin the nodes as long as the coupling strength is large enough, and the entire network can consensus exponentially fast. In many cases, some variables of the leader can only be estimated rather than directly measured. As a result, distributed estimation has become an important topic in the study of multi-agent systems, which has potential applications in the areas of sensor networks and robot networks. Recently, Hong *et al.* [23] designed a neighbour-based state-estimator for each autonomous agent to track the active leader with changing and unmeasurable states. Peng and Wang [24] also designed an estimator for each agent to estimate the leader's velocity in the presence of time-varying delays.

Compared with [17], this paper also investigates a second-order leader–follower consensus of non-linear multi-agent systems with all the agents, including the leader governed by the same more complicated intrinsic dynamics. The main differences between our paper and [17] include: First, here we suppose that an agent knows the position, but not the velocity, of the leader if they are connected, which is more realistic in many cases. Thus, we need to design a velocity estimator for each agent, based on which a pinning control algorithm is proposed to achieve leader–follower consensus without assuming that the interaction topology is strongly connected or contains a directed spanning tree, and it is proved that the velocity tracking errors of the estimators converge to zero asymptotically. Secondly, we consider the switching topologies between the leader and followers, and then a multiple Lyapunov function (MLF) is constructed to prove that each agent can follow the leader by this control scheme, if the dwell time τ is large enough. And thirdly, we provide lower bounds for the number of the pinning agents and the dwell time, respectively, in order to reach consensus.

The remainder of the paper is organised as follows: In Section 2, some basic definitions and preliminary results in graph theory are provided. In Section 3, the distributed estimators are designed and the pinning control scheme is proposed for non-linear multi-agent systems. In Section 4, some new results on the second-order consensus problem are obtained, while these results are validated by the simulation experiments in Section 5. Finally, the paper is concluded in Section 6.

Notations: Specifically, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{-1}$ denote transpose and inverse, respectively. $I_N \in R^{N\times N}$ is the identity matrix, and $1_N = (1, \ldots, 1)^{\mathrm{T}} \in R^N$. $\otimes$ denotes the Kronecker product. For $A \in R^{N\times N}$, $A > 0$ $(A \geq 0)$ if A is positive definite (positive semi-definite), and $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are the maximum and minimum eigenvalues of the matrix A, respectively. For two real symmetric matrices X and Y, $X > Y (X \geq Y)$ means that $X - Y > 0 (X - Y \geq 0)$.

2 Preliminaries

First, some basic definitions in the graph theory are introduced for subsequent use.

In general, information exchange between agents in a multi-agent system can be modelled by directed or undirected graphs [3, 25]. Let $\Re = (V, \zeta, A)$ be a weighted directed graph of N agents with a set of nodes $V = \{\pi_1, \pi_2, \ldots, \pi_N\}$, a set of edges $\zeta \subseteq V \times V$, and a non-negative adjacency matrix $A = [a_{ij}]$, which are used to represent the communication topology. The node indexes belong to a finite index set $I = \{1, 2, \ldots, N\}$. An edge of $\Re$ is denoted by $e_{ij} = (\pi_i, \pi_j)$. The adjacency elements associated with the edges of the graph are positive, that is, $\left(\pi_i, \pi_j\right) \in \zeta \Leftrightarrow a_{ij} > 0$, and the neighbour set of node π_i is denoted by $N_i = \{\pi_j | (\pi_j, \pi_i) \in \zeta\}$. Moreover, we assume that $a_{ii} = 0$ for all $i \in I$, which means the graph has no self-loop. The Laplacian matrix $L = [l_{ij}] \in R^{N\times N}$ associated with the adjacency matrix A is defined as

$$\begin{aligned} l_{ij} &= -a_{ij}, \quad i \neq j \\ l_{ii} &= \sum_{j\in N_i} a_{ij} \end{aligned} \tag{1}$$

which ensures that $\sum_{j=1}^{N} l_{ij} = 0$, $\forall i \in I$.

A directed path in a graph from π_i to π_j is a sequence of distinct vertices starting with π_i and ending with π_j such that consecutive vertices are adjacent. A directed graph is strongly connected, if for any two distinct nodes π_j and π_i, there exists a directed path from node π_j to node π_i. A directed graph has a directed spanning tree if there exists at least one node called root, which has a directed path to all the other nodes.

The following lemmas will be used in the derivation of the main results.

Lemma 1 [26]: Consider a symmetric matrix $D = \left[\begin{smallmatrix} A & E \\ E^{\mathrm{T}} & C \end{smallmatrix}\right]$, where $A = A^{\mathrm{T}}$, and $C = C^{\mathrm{T}}$, then D is positive-definite if and only if both C and $A - EC^{-1}E^{\mathrm{T}}$ are positive-definite.

Lemma 2 [27]: Suppose that the eigenvalues of symmetric matrices A and $B \in R^{m\times m}$ satisfy $\lambda_1(A) \leq \lambda_2(A) \leq \cdots \leq \lambda_m(A)$, and $\lambda_1(B) \leq \lambda_2(B) \leq \cdots \leq \lambda_m(B)$, then the following inequalities hold: $\lambda_i(A) + \lambda_1(B) \leq \lambda_i(A + B) \leq \lambda_i(A) + \lambda_m(B)$, $\forall i = 1, 2, \ldots, m$.

Lemma 3 [21, 28]: For a symmetric matrix $M \in R^{N\times N}$ and a diagonal matrix $D = \mathrm{diag}\left(d_1, \ldots, d_l, \underbrace{0, \ldots, 0}_{N-l}\right)$ with $d_i > 0$, $i = 1, \ldots, l$, let $M - D = \left[\begin{smallmatrix} A-\tilde{D} & B \\ B^{\mathrm{T}} & M_l \end{smallmatrix}\right]$, where M_l is the principal matrix of M by removing its first l row–column pairs, A and B are matrices with appropriate dimensions, and $\tilde{D} = \mathrm{diag}\,(d_1, \ldots, d_l)$. If $d_i > \lambda_{\max}\left(A - BM_l^{-1}B^{\mathrm{T}}\right)$, $i = 1, \ldots, l, M - D < 0$ is equivalent to $M_l < 0$.

3 System model

In this section, a new pinning-controlled algorithm and distributed estimators are proposed for non-linear multi-agent systems.

Here, a non-linear multi-agent system is composed of N coupled autonomous agents with second-order dynamics

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = f\left(x_i(t), v_i(t)\right) - bv_i(t) + u_i(t), \quad \forall i \in I \end{cases} \tag{2}$$

where $x_i(t) \in R^n$ and $v_i(t) \in R^n$ denote the position and the velocity, respectively, $b > 0$ denotes the velocity damping gain indicating that $-bv_i$ represents the velocity damping term. The initial conditions of the system is set to be $x_i(0) = x_{i0}$, $v_i(0) = v_{i0}$. It is assumed that the damping force is in proportion to the magnitude of velocity. Besides, $u_i(t) \in R^m$ is the control input or the protocol in consensus problem, and $f(x_i, v_i) \in R^n$ is the intrinsic dynamic of agent i. Note that here all the agents and the leader share the same intrinsic dynamics for simplicity [29, 30].

A leader is described by a double integrator of the form

$$\begin{cases} \dot{x}_0(t) = v_0(t) \\ \dot{v}_0(t) = f\left(x_0(t), v_0(t)\right) - bv_0(t) \\ y(t) = x_0(t) \end{cases} \tag{3}$$

where $y(t) = x_0(t)$ is the only variable of the leader that can be measured directly by the agents when they are connected to the leader. In addition, its velocity $v_0(t)$ keeps changing and cannot be measured. The objective of second-order consensus with the leader is to design control input u_i, $i = 1, 2, \ldots, N$, such that for any initial condition, the agents can follow the leader in the sense of both position and velocity, that is, $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$, $\lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0$.

Since $v_0(t)$ cannot be measured even when the agents are connected to the leader, we have to estimate $v_0(t)$ by using the information obtained from its neighbours in a distributed way. The estimation of $v_0(t)$ by agent i is denoted by $\hat{v}_i(t) (i = 1, 2, \ldots, N)$. Thus, for each agent, a neighbour-based coupling rule, which consists of two parts, can be expressed as follows

$$u_i(t) = \underbrace{k \sum_{j\in N_i} a_{ij}\left[\left(x_j - x_i\right) + \left(v_j - v_i\right)\right]}_{\alpha_i} - \underbrace{d_i\left[\left(x_i - x_0\right) + \left(v_i - \hat{v}_i\right)\right]}_{\beta_i}, \quad i = 1, 2, \ldots, n \tag{4}$$

where $k > 0$, d_i is the pinning control gain, satisfying $d_i > 0$ if agent i is connected to the leader and $d_i = 0$ otherwise, α_i is the consensus term, which is used to regulate position and velocity between agent i and its neighbours, and β_i is the pinning control term, which is used to track the leader. Without loss of generality, rearrange the order of all the agents and let the first $l (1 \le l \le N)$ agents in multi-agent system (2) be connected.

A neighbour-based dynamics system to estimate $v_0(t)$ is designed as follows

$$\begin{aligned} \dot{\hat{v}}_i(t) = {} & f\left(x_i(t), \hat{v}_i(t)\right) - b\hat{v}_i(t) \\ & + k\sum_{j\in N_i} a_{ij}\left[\left(x_j - x_i\right) + \left(\hat{v}_j - \hat{v}_i\right)\right] \\ & - 2d_i\left(x_i - x_0\right) - d_i\left(v_i - \hat{v}_i\right), \quad i = 1, 2, \ldots, N \end{aligned} \tag{5}$$

Remark 1: Note that the distributed estimator is the first-order dynamics system, which is helpful to construct a MLF of switching topologies between leader and followers. Note that, from (5), $\hat{v}_i(t)$ depends on the information from its neighbours $\hat{v}_j(t)$ and the measured output of the leader. When $i = l + 1, \ldots, N$, we have the pinning control gain $d_i = 0$, thus the dynamic system of $\hat{v}_i(t)$ is equivalent to the dynamic system of $v_i(t)$.

4 Main results

In this section, the consensus problem for non-linear multi-agent systems (2) with the pinning-controlled algorithms (4) is investigated. The convergence analysis is presented and algebraic criteria are established for the leader–follower consensus problem. Furthermore, we discuss switching topologies between leader and followers as an extension. The following assumption is needed for the derivation of the main results in this paper.

Assumption 1: For the non-linear function $f = (f_1, \ldots, f_n)^{\mathrm{T}}$ in (2), there exist two constant matrices $W = [w_{ij}] \in R^{n\times n}$ and $M = [m_{ij}] \in R^{n\times n}$, in which w_{ij}, $m_{ij} \ge 0$, such that

$$\left|f_i(x, v) - f_i(y, z)\right| \le \sum_{j=1}^{n}\left[w_{ij}\left|x_j - y_j\right| + m_{ij}\left|v_j - z_j\right|\right], \quad \forall i = 1, 2, \ldots, n, x, y, v, z \in R^n \tag{6}$$

Note that Assumption 1 is a Lipschitz-like condition, which is satisfied by, for example, all piecewise linear functions [31].

According to two matrices W and M, define $p = (1/2)\max_{1\le j\le n}\left\{\sum_{k=1}^{n}\left(w_{jk}^{2\xi} + m_{jk}^{2\xi} + 2w_{kj}^{2(1-\xi)}\right)\right\}$, $q = (1/2)\max_{1\le j\le n}\left\{\sum_{k=1}^{n}\left(w_{jk}^{2\xi} + m_{jk}^{2\xi} + 2m_{kj}^{2(1-\xi)}\right)\right\}$ and $c = (1/2)\max_{\le 1j\le n}\left\{\sum_{k=1}^{n}\left(m_{jk}^{2\xi} + m_{kj}^{2(1-\xi)}\right)\right\}$, where $\xi \in [0, 1]$. Construct the symmetric matrix $H = -k((L + L^T)/2)) + \rho I_N$, and $\rho = \max\{p, q + 1\}$. Let $D = \mathrm{diag}\left(d_1, \ldots d_l, \underbrace{0, \ldots 0}_{N-l}\right)$, where d_i is the pinning control gain. By matrix decomposition, one has $H - D = \begin{bmatrix} \tilde{A} - \tilde{D} & B \\ B^{\mathrm{T}} & H_l \end{bmatrix}$, where $H_l = -k((L + L^T)/2)_l + \rho I_l$ is the principal matrix of H by removing its first l row–column pairs, $\tilde{A}$ and B are matrices with appropriate dimensions, and $\tilde{D} = \mathrm{diag}\left(d_1, \ldots d_l\right)$.

The main result of this paper is given by the following theorem:

Theorem 1: Given system (2)–(5), suppose that Assumption 1 holds. If the following conditions

$$\lambda_{\min}\left\{\left(\frac{L + L^{\mathrm{T}}}{2}\right)_l\right\} > \frac{\rho}{k} \tag{7}$$

$$\rho - p + d_i > \lambda_{\max}\left(\tilde{A} - BH_l^{-1}B^{\mathrm{T}}\right), \quad \forall i = 1, \ldots, l \tag{8}$$

$$\rho - q - 1 + b - \frac{d_i}{2} > \max\left\{0, \lambda_{\max}\left(\tilde{A} - BH_l^{-1}B^{\mathrm{T}}\right)\right\}, \quad \forall i = 1, \ldots, l \tag{9}$$

$$\rho - \frac{1}{2} + \frac{b}{2} + \frac{d_i}{2} > \lambda_{\max}\left(\tilde{A} - BH_l^{-1}B^{\mathrm{T}}\right), \quad \forall i = 1, \ldots, l \tag{10}$$

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are satisfied, then

$$\lim_{t\to\infty}\|x_i(t)-x_0(t)\| = \lim_{t\to\infty}\|v_i(t)-v_0(t)\| = \lim_{t\to\infty}\|\hat{v}_i(t)-v_0(t)\| = 0$$

Proof: By taking $\bar{x} = x - 1_N \otimes x_0$, $\bar{v} = v - 1_N \otimes v_0$, $\tilde{v} = \hat{v} - v$, where $x = (x_1^{\mathrm{T}}, x_2^{\mathrm{T}}, \ldots, x_N^{\mathrm{T}})^{\mathrm{T}}$ $v = (v_1^{\mathrm{T}}, v_2^{\mathrm{T}}, \ldots, v_N^{\mathrm{T}})^{\mathrm{T}}$ and $\hat{v} = (\hat{v}_1^{\mathrm{T}}, \hat{v}_2^{\mathrm{T}}, \ldots, \hat{v}_N^{\mathrm{T}})^{\mathrm{T}}$. Then, the error dynamics of (2)–(5) can be written as (see (11))

where $F(x, v) = [f^T(x_1, v_1), \ldots, f^T(x_N, v_N)]^{\mathrm{T}}$ and $F(x, v) = [f^{\mathrm{T}}(x_1, \hat{v}_1), \ldots, f^{\mathrm{T}}(x_N, \hat{v}_N)]^{\mathrm{T}}$.

Let $\hat{y} = [\bar{x}^{\mathrm{T}}, \bar{v}^{\mathrm{T}}, \tilde{v}^{\mathrm{T}}]^{\mathrm{T}} \in R^{3nN}$, consider the following auxiliary function

$$V(t) = \frac{1}{2}\hat{y}^{\mathrm{T}}(P \otimes I_n)\hat{y} \tag{12}$$

where

$$P = \begin{bmatrix} k(L+L^{\mathrm{T}}) + D + bI_N & I_N & 0 \\ I_N & I_N & 0 \\ 0 & 0 & I_N \end{bmatrix}$$

First, we show that the matrix $P > 0$. Consider the auxiliary matrix $H - \hat{D}$, where $\hat{D} = \left[\begin{smallmatrix} \hat{D}_l & 0 \\ 0 & 0 \end{smallmatrix}\right]$, and $\hat{D}_l = \mathrm{diag}\{\rho - (1/2) + (b/2) + (d_1/2), \ldots, \rho - (1/2) + (b/2) + (d_l/2)\}$. Using matrix decomposition, one has $H - \hat{D} = \left[\begin{smallmatrix} \tilde{A}-\hat{D}_l & B \\ B^{\mathrm{T}} & H_l \end{smallmatrix}\right]$. It follows from (7) that $-k\lambda_{\min}\{((L+L^T)/2)_l\} + \rho < 0$. Therefore, from Lemma 2, $\lambda_{\max}(H_l) < 0$, that is, $H_l < 0$. With the definition of ρ, one has $\rho - (1/2) + (b/2) + (d_i/2) > 0$, $\forall i = 1, \ldots, l$. By (10) and Lemma 3, $H - \hat{D} < 0$. Meanwhile

$$\begin{aligned} -k\left(\frac{L+L^{\mathrm{T}}}{2}\right) + \frac{1}{2}I_N - \left(\frac{b}{2}I_N + \frac{D}{2}\right) &= -k\left(\frac{L+L^{\mathrm{T}}}{2}\right) + \rho I_N - \left(\rho I_N - \frac{1}{2}I_N + \frac{b}{2}I_N + \frac{D}{2}\right) \\ &\le -k\left(\frac{L+L^{\mathrm{T}}}{2}\right) + \rho I_N - \hat{D} \\ &= H - \hat{D} < 0 \end{aligned} \tag{13}$$

Therefore it is satisfied that

$$k(L+L^{\mathrm{T}}) + (b-1)I_N + D > 0 \tag{14}$$

According to Lemma 1, $kt(L+L^{\mathrm{T}}) + (b-1)I_N + D > 0$ is equivalent to $P > 0$. Hence, the auxiliary function $V(t)$ is a Lyapunov function.

Next, in order to study the behaviour of $\dot{V}(t)$ for $t \in [0, +\infty)$, we can write (see (15))

Consider the term $\bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)]$, where $\bar{x} = [\bar{x}_1^{\mathrm{T}}, \ldots, \bar{x}_n^{\mathrm{T}}]^{\mathrm{T}}$, $\bar{x}_i = [\bar{x}_{i1}, \ldots, \bar{x}_{in}]^{\mathrm{T}}$, and $F(x,v) - 1_N \otimes f(x_0, v_0) = [f^{\mathrm{T}}(x_1, v_1) - f^{\mathrm{T}}(x_0, v_0), \ldots, f^{\mathrm{T}}(x_N, v_1) - f^{\mathrm{T}}(x_N, v_0)]^{\mathrm{T}}$, thus

$$\begin{aligned} \bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)] &= \sum_{i=1}^{N} \bar{x}_i^{\mathrm{T}}[f(x_i, v_i) - f(x_0, v_0)] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{n} \bar{x}_{ij}[f_j(x_i, v_i) - f_j(x_0, v_0)] \end{aligned} \tag{16}$$

where $f(x_i, v_i) - f(x_0, v_0) = [f_1(x_i, v_i) - f_1(x_0, v_0), \ldots, f_n(x_i, v_i) - f_n(x_0, v_0)]^{\mathrm{T}}$.

According to Assumption 1, one obtains that

$$\begin{aligned} &\left|\bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)]\right| \\ &= \left|\sum_{i=1}^{N}\sum_{j=1}^{n} \bar{x}_{ij}\left[f_j(x_i, v_i) - f_j(x_0, v_0)\right]\right| \\ &\le \sum_{i=1}^{N}\sum_{j=1}^{n} |\bar{x}_{ij}|\left\{\sum_{k=1}^{n} w_{jk}|x_{ik} - x_{0k}| + \sum_{k=1}^{n} m_{jk}|v_{ik} - v_{0k}|\right\} \\ &= \sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{jk}|\bar{x}_{ij}||\bar{x}_{ik}| + \sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}|\bar{x}_{ij}||\bar{v}_{ik}| \end{aligned} \tag{17}$$

Owing to the fact that $2a|xy| \le a^{2\xi}x^2 + a^{2(1-\xi)}y^2$, $\forall a > 0$, $x, y \in R$, $\xi \in [0, 1]$, (17) can be further rewritten as

$$\begin{aligned} &\left|\bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)]\right| \\ &\le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{jk}^{2\xi}\bar{x}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{jk}^{2(1-\xi)}\bar{x}_{ik}^2 \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2\xi}\bar{x}_{ij}^2 \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2(1-\xi)}\bar{v}_{ik}^2 \end{aligned} \tag{18}$$

$$\begin{cases} \dot{\bar{x}} = \bar{v} \\ \dot{\bar{v}} = F(x,v) - 1_N \otimes f(x_0, v_0) - [(kL + D) \otimes I_n]\bar{x} - [(kL + bI_N) \otimes I_n]\bar{v} + (D \otimes I_n)\tilde{v} \\ \dot{\tilde{v}} = F(x,\hat{v}) - F(x,v) - [(kL + bI_N) \otimes I_n]\tilde{v} - (D \otimes I_n)\bar{x} \end{cases} . \tag{11}$$

$$\begin{aligned} \dot{V}(t) = {} & \bar{x}^{\mathrm{T}}\left[\left(k\left(L+L^{\mathrm{T}}\right) + D + bI_N\right) \otimes I_n\right]\bar{v} + \bar{v}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)] - \bar{v}^{\mathrm{T}}((kL + D) \otimes I_n)\bar{x} \\ & - \bar{v}^{\mathrm{T}}((kL + bI_N) \otimes I_n)\bar{v} + \bar{v}^{\mathrm{T}}(D \otimes I_n)\tilde{v} + \tilde{v}^{\mathrm{T}}\left[F(x,\hat{v}) - F(x,v)\right] - \tilde{v}^{\mathrm{T}}((kL + bI_N) \otimes I_n)\tilde{v} \\ & - \tilde{v}^{\mathrm{T}}(D \otimes I_n)\bar{x} + \bar{v}^{\mathrm{T}}\bar{v} + \bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)] - \bar{x}^{\mathrm{T}}((kL + D) \otimes I_n)\bar{x} \\ & - \bar{x}^{\mathrm{T}}((kL + bI_N) \otimes I_n)\bar{v} + \bar{x}^{\mathrm{T}}(D \otimes I_n)\tilde{v} \\ = {} & \bar{x}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)] - \bar{x}^{\mathrm{T}}((kL + D) \otimes I_n)\bar{x} + \bar{v}^{\mathrm{T}}\bar{v} + \bar{v}^{\mathrm{T}}[F(x,v) - 1_N \otimes f(x_0, v_0)] \\ & - \bar{v}^{\mathrm{T}}((kL + bI_N) \otimes I_n)\bar{v} + \tilde{v}^{\mathrm{T}}\left[F(x,\hat{v}) - F(x,v)\right] - \tilde{v}^{\mathrm{T}}((kL + bI_N) \otimes I_n)\tilde{v} + \bar{v}^{\mathrm{T}}(D \otimes I_n)\tilde{v} \end{aligned} \tag{15}$$

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Similarly

$$\left|\bar{v}^{\mathrm{T}}\left[F(x,v)-1_N\otimes f\left(x_0,v_0\right)\right]\right| \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{jk}^{2\xi}\bar{v}_{ij}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} w_{jk}^{2(1-\xi)}\bar{x}_{ik}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2\xi}\bar{v}_{ij}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2(1-\xi)}\bar{v}_{ik}^2 \quad (19)$$

$$\left|\tilde{v}^{\mathrm{T}}\left[F(x,v)-F(x,\hat{v})\right]\right| \le \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2\xi}\tilde{v}_{ij}^2+\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\sum_{k=1}^{n} m_{jk}^{2(1-\xi)}\tilde{v}_{ik}^2 = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{n}\left\{\sum_{k=1}^{n}(m_{jk}^{2\xi}+m_{kj}^{2(1-\xi)})\right\}\tilde{v}_{ij}^2 \quad (20)$$

By (18)–(20) and the fact that $\bar{v}^{\mathrm{T}}(D\otimes I_n)\tilde{v}\le(1/2)\bar{v}^{\mathrm{T}}(D\otimes I_n)\bar{v}+(1/2)\tilde{v}^{\mathrm{T}}(D\otimes I_n)\tilde{v}$, we have

$$\begin{aligned}\dot{V}(t) &\le \bar{x}^{\mathrm{T}}\left[(-kL+pI_N-D)\otimes I_n\right]\bar{x} \\ &\quad+\bar{v}^{\mathrm{T}}\left[\left(-kL+(1-b+q)I_N+\frac{D}{2}\right)\otimes I_n\right]\bar{v} \\ &\quad+\tilde{v}^{\mathrm{T}}\left[\left(-kL+(c-b)I_N+\frac{D}{2}\right)\otimes I_n\right]\tilde{v} \\ &= \bar{x}^{\mathrm{T}}\left[\left(-k\frac{L+L^{\mathrm{T}}}{2}+pI_N-D\right)\otimes I_n\right]\bar{x} \\ &\quad+\bar{v}^{\mathrm{T}}\left[\left(-k\frac{L+L^{\mathrm{T}}}{2}+(1-b+q)I_N+\frac{D}{2}\right)\otimes I_n\right]\bar{v} \\ &\quad+\tilde{v}^{\mathrm{T}}\left[\left(-k\frac{L+L^{\mathrm{T}}}{2}+(c-b)I_N+\frac{D}{2}\right)\otimes I_n\right]\tilde{v} \quad (21)\end{aligned}$$

With the conditions (7) and (8), we obtain $-k((L+L^T)/2)+pI_N-D<0$. In fact, consider the auxiliary matrix $H-\hat{D}$, where $\hat{D}=\left[\begin{smallmatrix}\hat{D}_l & 0\\ 0 & 0\end{smallmatrix}\right]$, and $\hat{D}_l=\mathrm{diag}\{\rho-p+d_1,\ldots,\rho-p+d_l\}$. It follows from (7) and (8) that $H-\hat{D}<0$. Thus

$$\begin{aligned}&-k\frac{L+L^{\mathrm{T}}}{2}+pI_N-D \\ &\quad=-k\frac{L+L^{\mathrm{T}}}{2}+\rho I_N-[(\rho-p)I_N+D] \\ &\quad\le H-\hat{D}<0 \quad (22)\end{aligned}$$

Similarly, it follows from (7) and (9) that

$$-k\frac{L+L^{\mathrm{T}}}{2}+(1-b+q)I_N+\frac{D}{2}<0 \quad (23)$$

With the definition of q and c, we have $q>c$, which implies that

$$\begin{aligned}&-k\frac{L+L^{\mathrm{T}}}{2}+(c-b)I_N+\frac{D}{2} \\ &\quad\le -k\frac{L+L^{\mathrm{T}}}{2}+(1-b+q)I_N+\frac{D}{2}<0 \quad (24)\end{aligned}$$

In view of (21), we have $\dot{V}(t)<0$ if $\hat{y}\neq 0$. Consequently, the agents can follow the leader in the sense of both position and velocity, and the velocity tracking errors of estimators converge to zero asymptotically, that is, $\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=\lim_{t\to\infty}\|v_i(t)-v_0(t)\|=\lim_{t\to\infty}\|\hat{v}_i(t)-v_0(t)\|=0$. This completes the proof. □

Remark 2: Let us take a closer look at the conditions in Theorem 1. First, conditions (7)–(10) imply that the number of pinned agents should be equal or greater than 1, and less than N, that is, $1\le l<N$. In fact, if $l=N$, then $((L+L^T)/2)_N$ is an empty set, hence condition (7) is not satisfied. Moreover, if $l=0$, then H_N is an empty set, and H_N^{-1} does not exist, hence conditions (8)–(10) are not satisfied. Furthermore, from (8) to (10), it is easy to see that if the velocity damping gain b is large enough, then conditions (8)–(10) are satisfied trivially. However, for large b, one may find that the leader's velocity converges to zero rapidly according to (3), and it seems that the neighbour-based estimator design approach does not make sense. Meanwhile, as l increases, the magnitude of $\lambda_{\max}\left(\tilde{A}-BH_l^{-1}B^{\mathrm{T}}\right)$ decreases, and so does b. Thus, the number of pinned agents should be increased, which increases the cost in reality. So there is a tradeoff between l and b.

Remark 3: First, we give the definition of zero in-degree strong connected component. A strong connected component ς_k is considered to be zero in-degree if the neighbour set N_i of each agent i in ς_k satisfies $N_i\subseteq\varsigma_k$. Since the interaction topology of the followers does not have a directed spanning tree in this paper, the network of the followers must have multiple zero in-degree strong connected components, denoted by $\varsigma_1,\ldots,\varsigma_K$. Then, according to condition (7), we have $l\ge K$. In fact, suppose that $l<K$, there must be a zero in-degree strong connected component ς_j, such that there is no path from the leader to ς_j. Let L_j denote the Laplacian matrix of ς_j, considering the matrix $((L+L^T)/2)_l$, by rearranging the order of the agents, we have $\left((L+L^{\mathrm{T}})/2\right)_l=\left[\begin{smallmatrix}* & 0\\ 0 & ((L_j+L_j^{\mathrm{T}})/2)\end{smallmatrix}\right]$. Owing to the fact that $1^{\mathrm{T}}((L_j+L_j^{\mathrm{T}})/2)1=0$, one has the matrix $((L+L^T)/2)_l$ is not positive-definite. However, according to condition (7), it follows that the matrix $((L+L^T)/2)_l$ is positive-definite, which results in a contradiction.

Note that in Theorem 1, the algebraic criteria (7)–(10) are established to achieve leader–follower consensus of non-linear multi-agent systems (2)–(5). However, the conditions (8)–(10) seem to be difficult to implement, because both sides of the inequalities are the functions of ρ. Based on Theorem 1 and its proof, we have the following corollary to resolve this problem.

Corollary 1: Given system (2)–(5), suppose that Assumption 1 holds. If the following conditions are satisfied

$$\lambda_{\min}\left\{\left(\frac{L+L^{\mathrm{T}}}{2}\right)_l\right\}>\frac{\rho}{k} \quad (25)$$

$$d_i>\lambda_{\max}\left(\tilde{A}-BH_l^{-1}B^{\mathrm{T}}\right),\quad \forall i=1,\ldots,l \quad (26)$$

$$b-\frac{d_i}{2}>\max\left\{0,\lambda_{\max}\left(\tilde{A}-BH_l^{-1}B^{\mathrm{T}}\right)\right\},\quad \forall i=1,\ldots,l \quad (27)$$

then

$$\lim_{t\to\infty}\|x_i(t)-x_0(t)\|=\lim_{t\to\infty}\|v_i(t)-v_0(t)\| =\lim_{t\to\infty}\left\|\hat{v}_i(t)-v_0(t)\right\|=0$$

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In practice, the relationships between leader and followers may vary over time, and their interconnection topology may also be dynamically changing. Consequently, switching topologies between leader and followers should be taken into account.

Suppose that there is an infinite sequence of bounded, non-overlapping, contiguous time-intervals $[t_i, t_{i+1})$, $i = 0, 1, \ldots$ with $t_0 = 0$. To avoid infinite switching during a finite time interval, assume as usual that there is a constant τ (called dwell time) with $t_{j+1} - t_j \geq \tau$ for all $j \geq 0$. Denote $\wp = \{D_1, D_2, \ldots, D_m\}$ as a set of the pinning control gain matrices of all possible topologies between leader and followers, and $\Im = \{1, 2, \ldots, m\}$ as its index set. Without loss of generality, we assume that $d_i(t) = \bar{d}$ if agent i is connected to the leader at t and $d_i(t) = 0$ otherwise, where $\bar{d}$ is a constant. For each D_k, there exists an orthogonal matrix U_k, such that $\breve{D}_k = U_k^{\mathrm{T}} D_k U_k = \mathrm{diag}\left(\underbrace{\bar{d}, \ldots, \bar{d}}_{l_k}, 0, \ldots, 0\right)$. The piecewise-constant function $\sigma(t) : [0, \infty) \to \Im$, is the switching signal, which is used to describe the variable interconnection topology between leader and followers. Moreover, $\sigma(t) = k$ means that the pinning control gain matrix D_k is activated at time t. Therefore the corresponding error dynamics system with switching topologies can be written as (see (28))

Next, consider the switching topologies, we have the following result, which is an extension of Theorem 1.

Theorem 2: Given system (28), suppose that Assumption 1 holds, and the following conditions are satisfied

$$\lambda_{\min}\left\{\left(\frac{L_i + L_i^{\mathrm{T}}}{2}\right)_{l_i}\right\} > \frac{\rho}{k}, \quad \forall i = 1, \ldots, m \tag{29}$$

$$\rho - p + \bar{d} > \lambda_{\max}\left(\tilde{A}_i - B_i H_{l_i}^{-1} B_i^{\mathrm{T}}\right), \quad \forall i = 1, \ldots, m \tag{30}$$

$$\rho - q - 1 + b - \frac{\bar{d}}{2} > \max\left\{0, \lambda_{\max}\left(\tilde{A}_i - B_i H_{l_i}^{-1} B_i^{\mathrm{T}}\right)\right\}, \quad \forall i = 1, \ldots, m \tag{31}$$

$$\rho - \frac{1}{2} + \frac{b}{2} + \frac{\bar{d}}{2} > \lambda_{\max}\left(\tilde{A}_i - B_i H_{l_i}^{-1} B_i^{\mathrm{T}}\right), \quad \forall i = 1, \ldots, m \tag{32}$$

where $\left(\left(L_i + L_i^{\mathrm{T}}\right)/2\right)_{l_i}$ is the minor matrix of $\left(\left(L_i + L_i^{\mathrm{T}}\right)/2\right)$ by removing its first l_i row–column pairs, $L_i = U_i^{\mathrm{T}} L U_i$, and $H_i = U_i^{\mathrm{T}} H U_i = -k\left(\left(L_i + L_i^{\mathrm{T}}\right)/2\right) + \rho I_N = \begin{bmatrix} \tilde{A}_i & B_i \\ B_i^{\mathrm{T}} & H_{l_i} \end{bmatrix}$. If the dwell time τ is large enough, then $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = \lim_{t\to\infty} \|v_i(t) - v_0(t)\| = \lim_{t\to\infty} \|\hat{v}_i(t) - v_0(t)\| = 0$.

Proof: Consider the following auxiliary function

$$V(t) = \frac{1}{2}\hat{y}^{\mathrm{T}}\left(P_\sigma \otimes I_n\right)\hat{y} \tag{33}$$

where

$$\hat{y} = \left[\bar{x}^{\mathrm{T}}, \bar{v}^{\mathrm{T}}, \tilde{v}^{\mathrm{T}}\right]^{\mathrm{T}} \in R^{3nN} \quad \text{and}$$

$$P_\sigma = \begin{bmatrix} k\left(L + L^{\mathrm{T}}\right) + D_\sigma + bI_N & I_N & 0 \\ I_N & I_N & 0 \\ 0 & 0 & I_N \end{bmatrix}$$

Take an interval $[t_i, t_{i+1})$ into consideration. Without loss of generality, assume that $\sigma(t) = i$ for $t \in [t_i, t_{i+1})$. Let

$$\tilde{y} = \left(\begin{bmatrix} U_i^{\mathrm{T}} & & \\ & U_i^{\mathrm{T}} & \\ & & U_i^{\mathrm{T}} \end{bmatrix} \otimes I_n\right)\hat{y} = \left[\tilde{y}_1^{\mathrm{T}}, \tilde{y}_2^{\mathrm{T}}, \tilde{y}_3^{\mathrm{T}}\right]^{\mathrm{T}}$$

thus

$$\begin{aligned} V(t) &= \frac{1}{2}\tilde{y}^{\mathrm{T}}\left(\begin{bmatrix} U_i^{\mathrm{T}} & & \\ & U_i^{\mathrm{T}} & \\ & & U_i^{\mathrm{T}} \end{bmatrix} \otimes I_n\right)\left(P_i \otimes I_n\right) \\ &\quad \times \left(\begin{bmatrix} U_i & & \\ & U_i & \\ & & U_i \end{bmatrix} \otimes I_n\right)\tilde{y} \\ &= \frac{1}{2}\tilde{y}^{\mathrm{T}}\left(\breve{P}_i \otimes I_n\right)\tilde{y} \end{aligned} \tag{34}$$

where

$$\breve{P}_i = \begin{bmatrix} k\left(L_i + L_i^{\mathrm{T}}\right) + \breve{D}_i + bI_N & I_N & 0 \\ I_N & I_N & 0 \\ 0 & 0 & I_N \end{bmatrix}$$

and $L_i = U_i^{\mathrm{T}} L U_i$. Similar to the analysis of Theorem 1, we have that

$$\begin{aligned} \dot{V}(t) \leq{}& \tilde{y}_1^{\mathrm{T}}\left[\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + pI_N - \breve{D}_i\right) \otimes I_n\right]\tilde{y}_1 \\ &+ \tilde{y}_2^{\mathrm{T}}\left[\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + (1 - b + q)I_N + \frac{\breve{D}_i}{2}\right) \otimes I_n\right]\tilde{y}_2 \\ &+ \tilde{y}_3^{\mathrm{T}}\left[\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + (c - b)I_N + \frac{\breve{D}_i}{2}\right) \otimes I_n\right]\tilde{y}_3 \end{aligned} \tag{35}$$

According to (29)–(32), we obtain that $\dot{V}(t) < 0$ for $t \in [t_i, t_{i+1})$ if $\tilde{y} \neq 0$, where $V(t)$ is called MLF [32, 33]. By Lemma 9 in [32], we have that the switched system (28) is globally asymptotically stable, that is, $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$, $\lim_{t\to\infty} \|v_i(t) - v_0(t)\| = 0$ and $\lim_{t\to\infty} \|\hat{v}_i(t) - v_0(t)\| = 0$, if the dwell time τ is large enough. Then the proof is complete. □

The following corollary is a generation of Corollary 1.

Corollary 2: Given system (28), suppose that Assumption 1 holds, and the following conditions are satisfied

$$\lambda_{\min}\left\{\left(\frac{L_i + L_i^{\mathrm{T}}}{2}\right)_{l_i}\right\} > \frac{\rho}{k}, \quad \forall i = 1, \ldots, m \tag{36}$$

$$\bar{d} > \lambda_{\max}\left(\tilde{A}_i - B_i H_{l_i}^{-1} B_i^{\mathrm{T}}\right), \quad \forall i = 1, \ldots, m \tag{37}$$

$$\begin{cases} \dot{\bar{x}} = \bar{v} \\ \dot{\bar{v}} = F(x, v) - 1_N \otimes f(x_0, v_0) - [(kL + D) \otimes I_n]\bar{x} - [(kL + bI_N) \otimes I_n]\bar{v} + (D_\sigma \otimes I_n)\tilde{v} \\ \dot{\tilde{v}} = F(x, \hat{v}) - F(x, v) - [(kL + bI_N) \otimes I_n]\tilde{v} - (D_\sigma \otimes I_n)\bar{x} \end{cases} \tag{28}$$

$$b - \frac{\bar{d}}{2} > \max\left\{0, \lambda_{\max}\left(\tilde{A}_i - B_i H_{l_i}^{-1} B_i^{\mathrm{T}}\right)\right\}, \quad \forall i = 1, \ldots, m \tag{38}$$

then it can be obtained that $\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = \lim_{t\to\infty}\|v_i(t) - v_0(t)\| = \lim_{t\to\infty}\left\|\hat{v}_i(t) - v_0(t)\right\| = 0$ if the dwell time τ is large enough.

In Theorem 2 and Corollary 2, it is shown that all the followers reach a consensus with the leader asymptotically provided that the dwell time τ is large enough, which leads to the question that how to determine a lower bound for τ. The following corollary tries to answer this question by giving a lower bound for τ to guarantee the consensus.

Denote

$$\lambda_1 = \max_{j\in\{1,\ldots,m\}}\left\{\lambda_{\max}(P_j)\right\}$$

$$\lambda_2 = \min_{j\in\{1,\ldots,m\}}\left\{\lambda_{\min}(P_j)\right\}$$

$$\lambda_3 = -\max_{j\in\{1,\ldots,m\}}\left\{\lambda_{\max}\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + pI_N - \breve{D}_i\right)\right.$$
$$\lambda_{\max}\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + (1 - b + q)I_N + \frac{\breve{D}_i}{2}\right)$$
$$\left.\lambda_{\max}\left(-k\frac{L_i + L_i^{\mathrm{T}}}{2} + (c - b)I_N + \frac{\breve{D}_i}{2}\right)\right\}$$

Corollary 3: Given system (28), suppose that Assumption 1 and (29)–(32) hold. If the dwell time τ satisfies the following inequality

$$\tau > \frac{\lambda_1[\ln\lambda_1 - \ln\lambda_2]}{2\lambda_3} \tag{39}$$

then

$$\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = \lim_{t\to\infty}\|v_i(t) - v_0(t)\| = \lim_{t\to\infty}\left\|\hat{v}_i(t) - v_0(t)\right\| = 0$$

Proof: According to Theorem 2, we have $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$. Take the interval $[0, t_1)$ into consideration, without loss of generality, assume that $\sigma(t) = \sigma_1$ for $t \in [0, t_1)$. It follows from (33) and (35) that

$$\dot{V}_{\sigma_1}(t) \le -\frac{2\lambda_3}{\lambda_1}V_{\sigma_1}(t) \tag{40}$$

namely

$$V_{\sigma_1}(t) \le \mathrm{e}^{-(2\lambda_3/\lambda_1)t}V_{\sigma_1}(0) \tag{41}$$

For $t \in [t_1, t_2)$, we have

$$\dot{V}_{\sigma_2}(t) \le -\frac{2\lambda_3}{\lambda_1}V_{\sigma_2}(t) \tag{42}$$

Therefore

$$\begin{aligned} V_{\sigma_2}(t) &\le \mathrm{e}^{-(2\lambda_3/\lambda_1)(t-t_1)}V_{\sigma_2}(t_1) \\ &\le \mathrm{e}^{-(2\lambda_3/\lambda_1)(t-t_1)}\frac{\lambda_1}{\lambda_2}V_{\sigma_1}(t_1) \\ &\le \mathrm{e}^{-(2\lambda_3/\lambda_1)t}\frac{\lambda_1}{\lambda_2}V_{\sigma_1}(0) \end{aligned} \tag{43}$$

By induction, for $t \in [t_i, t_{i+1})$, we obtain

$$V_{\sigma_{i+1}}(t) \le \mathrm{e}^{-(2\lambda_3/\lambda_1)t}\left(\frac{\lambda_1}{\lambda_2}\right)^i V_{\sigma_1}(0) \tag{44}$$

According to the definition of the dwell time τ, we have $i < (t/\tau)$, then (44) could be rewritten as

$$\begin{aligned} V_{\sigma_{i+1}}(t) &\le \mathrm{e}^{-(2\lambda_3/\lambda_1)t}\left(\frac{\lambda_1}{\lambda_2}\right)^{(t/\tau)}V_{\sigma_1}(0) \\ &= \mathrm{e}^{(-(2\lambda_3/\lambda_1)+(1/\tau)\ln(\lambda_1/\lambda_2))t}V_{\sigma_1}(0) \end{aligned} \tag{45}$$

As a result, in view of (39), all the agents can follow the leader in the sense of both position and velocity, and the velocity tracking errors of estimators converge to zero asymptotically, that is, $\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = \lim_{t\to\infty}\|v_i(t) - v_0(t)\| = \lim_{t\to\infty}\|\hat{v}_i(t) - v_0(t)\| = 0$. This completes the proof. □

5 Numerical simulation

In this section, two numerical examples are given to illustrate the effectiveness of the proposed theoretical results. In the simulation, the leader moves in a three-dimensional space, and its velocity is governed by the following Chua's oscillator in [34]

$$\begin{cases} \dot{v}_{01} = \theta\left(v_{02} - v_{01} - h\left(v_{01}\right)\right) - bv_{01} \\ \dot{v}_{02} = v_{01} - v_{02} + v_{03} - bv_{02} \\ \dot{v}_{03} = -\alpha\varepsilon\sin\left(\omega x_{01}\right) - \gamma v_{02} - \beta v_{03} - bv_{03} \end{cases} \tag{46}$$

where $v_0 = [v_{01}, v_{02}, v_{03}]^{\mathrm{T}}$, and $h(v_{01}) = b_0 v_{01} + 0.5(a_0 - b_0)[|v_{01} + 1| - |v_{01} - 1|]$. In order to verify whether the nonlinear function f in (46) satisfies Assumption 1, we first need to check if the function h satisfies the Lipschitz condition. In fact, for any x, $y \in R$, we have (see (47))

By some calculations, we obtain the following two matrices

$$W = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha\varepsilon\omega & 0 & 0 \end{bmatrix}$$
$$M = \begin{bmatrix} |\theta\,(1 + b_0)| + |\theta(a_0 - b_0)| & \theta & 0 \\ 1 & 1 & 1 \\ 0 & \gamma & \beta \end{bmatrix} \tag{48}$$

$$\begin{aligned} |h(x) - h(y)| &= \left|b_0 x + \frac{a_0 - b_0}{2}[|x + 1| - |x - 1|] - b_0 y - \frac{a_0 - b_0}{2}[|y + 1| - |y - 1|]\right| \\ &= \left|b_0(x - y) + \frac{a_0 - b_0}{2}[|x + 1| - |y + 1|] - \frac{a_0 - b_0}{2}[|x - 1| - |y - 1|]\right| \\ &\le |b_0(x - y)| + \left|\frac{a_0 - b_0}{2}\right||x + 1 - y - 1| + \left|\frac{a_0 - b_0}{2}\right||x - 1 - y + 1| \\ &= [|b_0| + |a_0 - b_0|]|x - y| \end{aligned} \tag{47}$$

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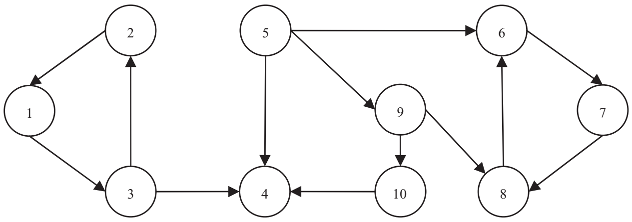

Fig. 1 *Interaction topology of ten agents*

and the parameters in (48) are chosen as $\theta = 1$, $\alpha = 13.52$, $\beta = 0.3698$, $\gamma = 1.376$, $\varepsilon = 0.2$, $w = 0.5$, $a_0 = -1.7523$, and $b_0 = -0.8345$. With the definition of p, q and ρ, by choosing $\xi = (1/2)$, we have $p = 2.3937$, $q = 4.876$, $\rho = 5.876$. Next, we consider ten agents with the interaction topology illustrated in Fig. 1.

Here, the interaction topology of these ten agents does not have a directed spanning tree. It is satisfied that $a_{ij} = 1$ for any $(\pi_i, \pi_j) \in \varsigma$, otherwise $a_{ij} = 0$. Then, the total error of the multi-agent system is defined as follows

$$
\begin{aligned}
e_x &= \frac{1}{10}\sum_{i=1}^{3}\sqrt{\sum_{j=1}^{10}\left[x_{ji}(t) - x_{0i}(t)\right]^2} \\
e_v &= \frac{1}{10}\sum_{i=1}^{3}\sqrt{\sum_{j=1}^{10}\left[v_{ji}(t) - v_{0i}(t)\right]^2} \\
e_{\hat{v}} &= \frac{1}{10}\sum_{i=1}^{3}\sqrt{\sum_{j=1}^{10}\left[\hat{v}_{ji}(t) - v_{0i}(t)\right]^2}
\end{aligned} \tag{49}
$$

Case 1: Fixed topology between leader and followers. At first, we should determine which agents should be pinned. From [28, 35], it is pointed out that for a diagraph $\Re$, let V and D denote the node set of $\Re$ and the pinned-node set, respectively. All nodes in V/D should have access to the pinned-node set D, that is, for any node $i \in V/D$, one can always find node $j \in D$ such that there exists a directed path from node j to node i. From Fig. 1, agent 5 should be pinned first, because its state is not affected by others. Furthermore, agents 1, 2 and 3 are strongly connected, and their states are not affected by others. Hence, agents 1, 2 and 3 should be chosen as pinned candidates. Based on comprehensive consideration of tracking performance and the number of pinning nodes, we choose agents 1, 2, 5 and 9 as pinned agents. The initial values of the leader's position and velocity are given by $x_0 = 11, 13, 12]^{\mathrm{T}}$ and $v_0 = [-1, 0, 1]^{\mathrm{T}}$, respectively. Furthermore, the initial positions, velocities of the ten agents and the initial velocities of the estimator are chosen randomly from the cube $[6, 20] \times [6, 20] \times [6, 20], [-0.1, 0.1] \times [-0.1, 0.1] \times [-0.1, 0.1], [-0.1, 0.1] \times [-0.1, 0.1] \times [-0.1, 0.1]$, respectively. Based on Theorem 1, we choose $k = 12.38$, $d_1 = d_2 = d_5 = d_9 = 1.9028$, and $b = 7.2302$. According to the dynamics (2)–(5), the evolutions of positions and velocities of these ten agents are shown in Figs. 2 and 3, respectively. It should be noted that the solid line represents the evolutions of the leader. Also, the evolution of velocity of estimators and the total error are shown in Figs. 4 and 5. From Figs. 2 to 5, one can see that the multi-agent system (2) with second-order dynamics reaches leader–follower consensus and the velocities of four estimators converge to the leader's velocity.

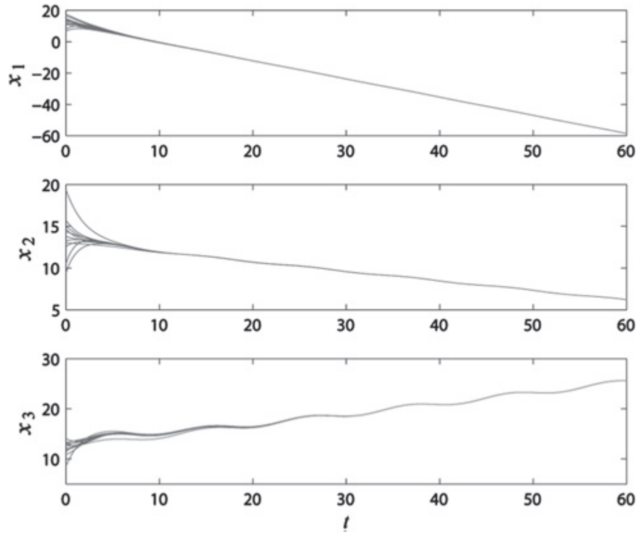

Fig. 2 *Positions of ten agents with fixed topology*

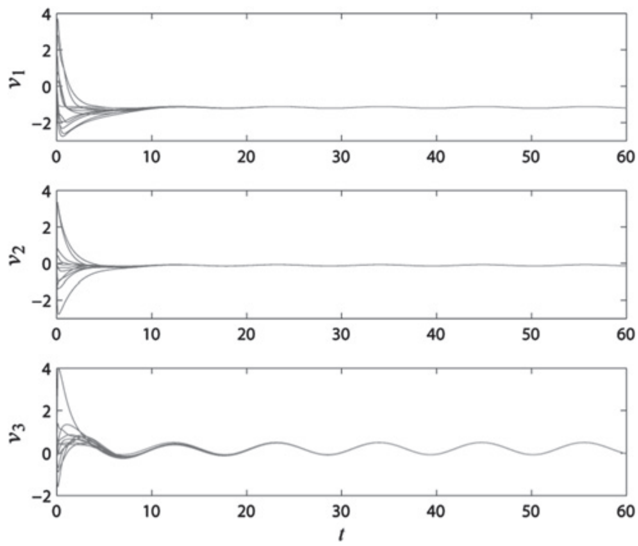

Fig. 3 *Velocities of ten agents with fixed topology*

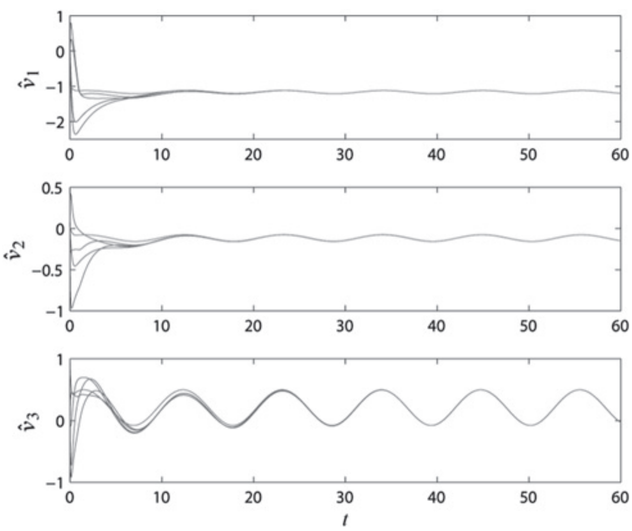

Fig. 4 *Velocities of four estimators with fixed topology*

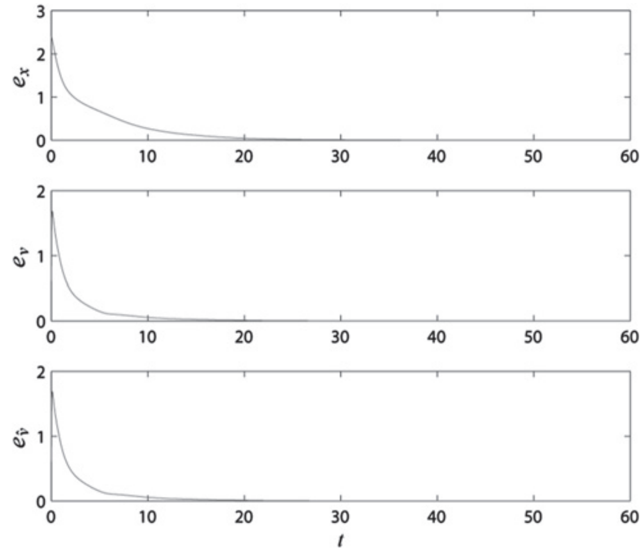

Fig. 5 *Evolution of the total error with fixed topology*

Case 2: Switching topologies between leader and followers. In this case, there are two pinning control gain matrices D_1 and D_2. For D_1, we choose agents 1, 5 and 9 as pinned agents, and for D_2, we choose agents 1, 2, 5 and 9 as pinned agents. The system (28) starts at D_1, and switches every $T = 10$ s between D_1 and D_2. Moreover, the initial values of leader's position and velocity are given by $x_0 = [11, 13, 12]^{\mathrm{T}}$ and $v_0 = [-1, 0, 1]^{\mathrm{T}}$, respectively. Also, the initial positions and velocities of the ten agents, and the velocities of the estimator are chosen randomly from the cube $[6, 20] \times [6, 20] \times [6, 20], [-0.1, 0.1] \times [-0.1, 0.1] \times [-0.1, 0.1]$ and $[-0.1, 0.1] \times [-0.1, 0.1] \times [-0.1, 0.1]$, respectively. Based on Theorem 2, we choose $k = 14.32$, $\bar{d} = 2.0141$ and $b = 7.3972$.

According to the dynamics (28), the evolutions of positions and velocities of these ten agents are shown in Figs. 6 and 7, respectively. Also, the evolutions of velocities of estimators and the total error are shown in Figs. 8 and 9. From Figs. 6–9, one can see that the multi-agent system (28) with second-order dynamics reaches leader–follower consensus

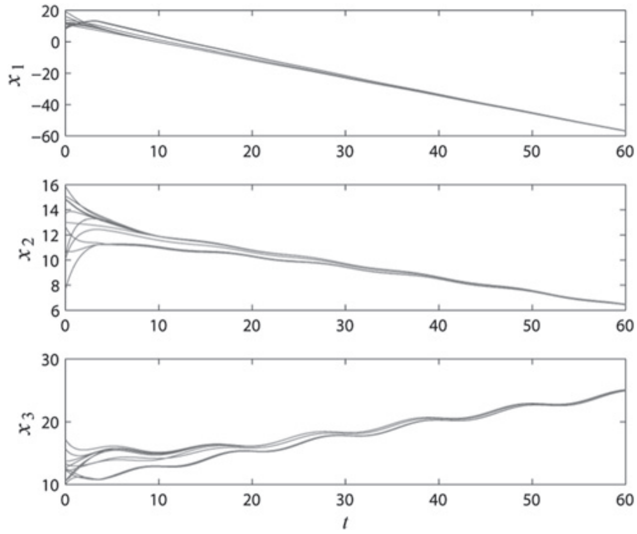

Fig. 6 *Positions of ten agents with switching topologies*

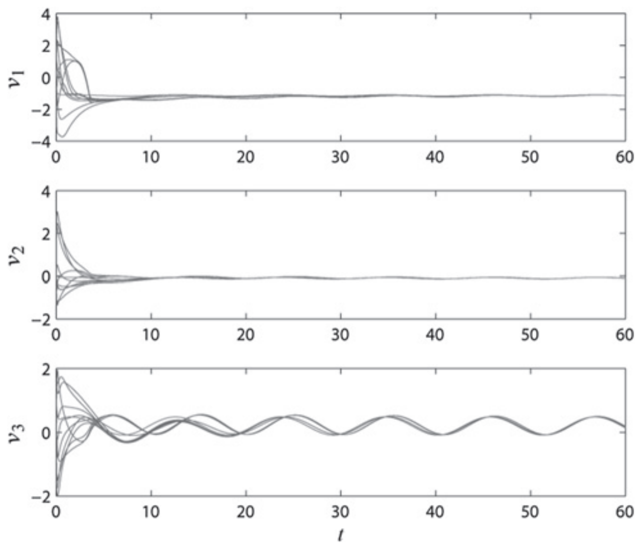

Fig. 7 *Velocities of ten agents with switching topologies*

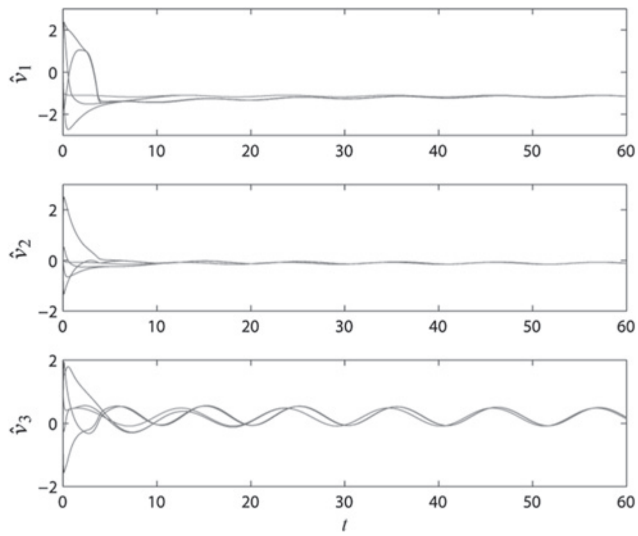

Fig. 8 *Velocities of four estimators with switching topologies*

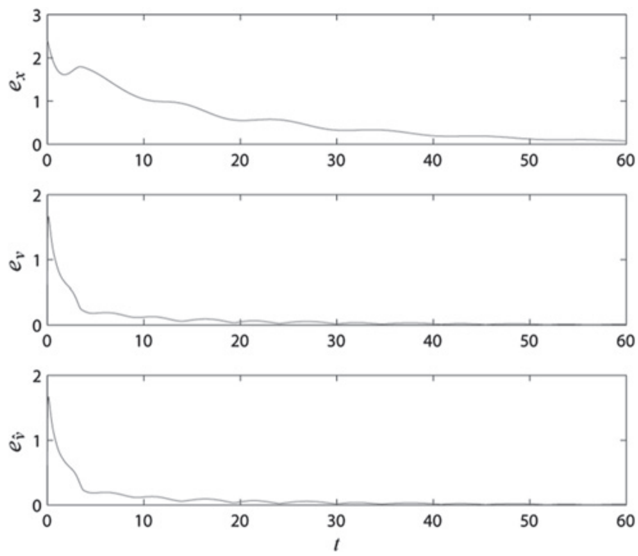

Fig. 9 *Evolution of the total error with switching topologies*

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and the velocities of four estimators converge to the leader's velocity.

6 Conclusion

This paper has discussed the consensus problem of a group of second-order agents with an active leader, whose velocity cannot be measured. To solve the problem, a neighbour-based estimator design approach and a pinning-controlled algorithm have been proposed for each autonomous agent. Without assuming that the interaction topology is strongly connected or contains a directed spanning tree, all the agents in the group could follow the leader, and the velocity tracking errors of estimators converged to zero asymptotically. Besides, the paper has also considered switching topologies between leader and followers. With the help of an MLF, it has been proved that if the dwell time τ is large enough, all the agents could also follow the leader in this case, and meanwhile, a lower bound for the dwell time is provided. Finally, two numerical simulations have been presented to illustrate the theoretical results. Further extensions will focus on the situations where the agents and the leader do not share the same intrinsic dynamics.

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