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Group consensus for heterogeneous multi-agent systems with parametric uncertainties

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ABSTRACT

In this paper, a group consensus problem is investigated for the heterogeneous agents that are governed by the Euler–Lagrange system and the double-integrator system, respectively, and the parameters of the Euler–Lagrange system are uncertain. To achieve group consensus, a novel group consensus protocol and a time-varying estimator of the uncertain parameters are proposed. By combining algebraic graph theory with the Barbalat lemma, several effective sufficient conditions are obtained. It is found that the time-delay group consensus can be achieved provided that the inner coupling matrices are equal in the different sub-networks. Besides, the switching topologies between homogeneous agents are also considered, with the help of the Barbalat-like lemma, and some relevant results are also obtained. Finally, these theoretical results are demonstrated by the numerical simulations.

1. Introduction

Over the past few years, extensive research has been conducted on distributed coordination of multi-agent systems from various scientific communities [1–8]. Applications of this research include formation control of mobile robots [9–12], design of sensor networks [13–16], optimization-based distributed control [17–20], and so on. Please be referred to [21,22] for a more comprehensive overview of the field.

As one type of distributed coordination problems, group consensus has many civil and military applications in surveillance, reconnaissance, battle field assessment, etc. Research on the group consensus not only helps better understand the mechanisms of natural collective phenomena, but also provides some useful ideas for distributed cooperative control. In the group consensus problem, the whole network is divided into multiple sub-networks with information exchanges between them, and the aim is to design appropriate protocol such that agents in the same sub-networks reach the same consistent states. In [23], Yu and Wang studied the group consensus problem in a multi-agent network with time-varying topologies, and introduced a double-tree-form transformation under which the dynamic equation of agents was transformed into a reduced-order system. In that work, it was assumed that the channels between different groups must exist continuously, thus the protocols proposed in [23] are purely continuous-time ones. Considering that the information exchange between different groups may be intermittent in practice, Hu et al. [24] investigated the group consensus problem with discontinuous information transmissions among different groups, and designed the hybrid protocol to solve it. Recently, Su et al. [25] considered the pinning control problem for cluster synchronization of undirected complex dynamical networks, and proposed a novel decentralized adaptive pinning-control scheme on both coupling strengths and feedback gains. Furthermore, in [26], Su et al. investigated the cluster synchronization of coupled harmonic oscillators with multiple leaders in an undirected fixed network, and it was shown that all oscillators in the same group could asymptotically synchronize with the corresponding leader even when only one oscillator in each group has access to the information of the corresponding leader.

Generally, the dynamics of agents is an important part of a multi-agent system, which will largely influence their consensus states. However, almost all the aforementioned results were only concerned with distributed coordination of homogeneous multi-agent systems, i.e. all the agents have the same dynamics, which might not be realistic in nature where individual heterogeneity is ubiquitous [27]. In [28], Zheng et al. considered the consensus problem of heterogeneous multi-agent system composed of first-order and second-order agents, for which the consensus protocols have both position and velocity information. It should be noted

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that all the agents achieve static consensus [29] asymptotically due to the existence of first-order agents. Based on the above results, the finite-time consensus problem of heterogeneous multi-agent systems was proposed in [30]. Liu and Hill [31] investigated the global consensus problem between a multi-agent system and a known objective signal by designing an impulsive consensus control scheme. In their framework, the consensus criteria in the multi-agent system with the uncertainties, heterogeneous agents, and coupling time-delays were studied in terms of linear matrix inequalities (LMIs) and algebraic inequalities. More recently, Liu et al. [32] studied the quasi-synchronization issue of linearly coupled networks with discontinuous nonlinear functions in each heterogeneous node. Here, quasi-synchronization means the synchronization with an error level. By introducing a virtual target, some sufficient quasi-synchronization conditions were presented and explicit expressions of error levels were derived to estimate the synchronization error in [33].

In this paper, we investigate a new group consensus of heterogeneous multi-agent systems, where two types of agents are grouped as two sub-networks. The agents in one sub-network are governed by the Euler–Lagrange system with parametric uncertainties, and those in another sub-network are described by the double-integrator dynamics. Meanwhile, there are time-delays when transmitting information between two sub-networks. Note that the group consensus protocol in [23] cannot be directly applied here due to the inherent nonlinearity of the Euler–Lagrange system. Instead, we design a novel group consensus protocol and a time-varying estimator of the uncertain parameters for the agents governed by the Euler–Lagrange system to achieve group consensus. Furthermore, considering that the interaction topology between agents may change dynamically in practice, we also investigate the group consensus with switching topologies. It is worth noting that there are such heterogeneous multi-agent systems in reality, such as the satellite-based vehicle positioning system in which the dynamics of satellite is described by the Euler–Lagrange system, while the dynamics of vehicle is governed by the double-integrator dynamics.

The rest of the paper is organized as follows. In Section II, some basic definitions in graph theory and mathematical preliminary results are provided. In Section III, a novel group consensus protocol and a time-varying estimator of the uncertain parameters are proposed. Some group consensus results are established, and these results are validated by the simulation experiments in Section IV. Finally, the paper is concluded in Section V.

Notations. Throughout this paper, $(\cdot)^T$ and $(\cdot)^{-1}$ denote transpose and inverse, respectively. $\otimes$ denotes the Kronecker product, and $\|\ \|$ is the Euclidean norm. If ω is a column vector, then $diag(\omega)$ denotes the diagonal matrix with the ith diagonal entry being the ith element of vector ω.

2. Preliminaries

In this section, some basic definitions in graph theory and preliminary mathematical results are firstly introduced for the subsequent use.

2.1. Topology description

For a multi-agent system, information exchange between agents can be modeled by directed or undirected graphs [2,22]. Let $G=(V,\zeta,A)$ be a weighted directed graph of n agents with a set of agents $V=\{\pi_1,\pi_2,\ldots\pi_n\}$, a set of edges $\zeta\subseteq V\times V$, and a non-negative adjacency matrix $A=[a_{ij}]$, which is used to represent the network topology. The agent indexes belong to a finite index set $I=\{1,2,\ldots n\}$, and an edge between two nodes is denoted by $e_{ij}=(\pi_i,\pi_j)$. Moreover, the adjacency elements associated with the edges of the graph are positive, i.e., $(\pi_i,\pi_j)\in\zeta\Leftrightarrow a_{ij}>0$, and the neighbor set of node π_i is denoted by $N_i=\{\pi_j|(\pi_j,\pi_i)\in\zeta\}$. Here, we assume that $a_{ii}=0$ for all $i\in I$, which means $i\notin N_i$ for each agent i. The elements in the corresponding Laplacian matrix $L=[l_{ij}]\in R^{n\times n}$ are defined as

$$l_{ij}=-a_{ij},\ i\neq j,\qquad l_{ii}=\sum_{j\in N_i}a_{ij},\tag{1}$$

satisfying that $\sum_{j=1}^{n}l_{ij}=0,\quad\forall i\in I$. A directed path in G from π_i to π_j is a sequence of distinct vertices starting from π_i and ending to π_j with the consecutive vertices being adjacent. Meanwhile, a directed graph is strongly connected, if there is a directed path between any two distinct nodes.

The following lemma gives a property of strongly connected graph.

Lemma 1. *([34]). Suppose that $G(A)$ represents a directed graph and L is the corresponding Laplacian matrix, then $G(A)$ is strongly connected if and only if there exists a positive column vector $\omega=[\omega_1,\cdots,\omega_n]^T$, such that $\omega^TL=0$. Furthermore, the matrix $((diag(\omega)L+L^Tdiag(\omega))/2)$ is positive semi-definite.*

2.2. System model

In our framework, there are two kinds of dynamics in the heterogeneous multi-agent systems.

Firstly, the agents in one sub-network are described by the Euler–Lagrange dynamics, which has the form

$$M_i(q_i)\ddot{q}_i+C_i(q_i,\dot{q}_i)q_i+g_i(q_i)=\tau_i,\quad i\in I_1=\{1,\ldots,n\},\tag{2}$$

where $q_i\in R^p$ is the vector of generalized coordinates, $M_i(q_i)\in R^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i,\dot{q}_i)q_i\in R^p$ is the vector of Coriolis and centrifugal torques, $g_i(q_i)$ is the vector of gravitational torque, and $\tau_i\in R^p$ is the group consensus protocol on the i th agent. Owing to the structure of Euler–Lagrange systems [35], Eq. (2) exhibits certain fundamental properties as follows:

(P1). For any $i\in\{1,\ldots,n+m\}$, there are positive constants $k_{\underline{M}}$, $k_{\overline{M}}$, $k_{\overline{C}}$, and $k_{\overline{g}}$ such that $k_{\underline{M}}I_p\le M_i(q_i)\le k_{\overline{M}}I_p$, $\|C_i(q_i,\dot{q}_i)\|\le k_{\overline{C}}\|\dot{q}_i\|$, and $\|g_i(q_i)\|\le k_{\overline{g}}$.

(P2). The matrix $\dot{M}_i(q_i)-2C_i(q_i,\dot{q}_i)$ is skew symmetric.

(P3). The Euler–Lagrange systems can be linearly parameterized, that is

$$M_i(q_i)x+C_i(q_i,\dot{q}_i)y+g_i(q_i)=Y_i(q_i,\dot{q}_i,x,y)\theta_i,\quad\forall x,y\in R^p,\tag{3}$$

where $Y_i(q_i,\dot{q}_i,x,y)$ is the regression matrix and θ_i is an unknown vector associated with the i th agent.

According to (P3), θ_i should be estimated by using the information from the regression matrix Y_i.

Then, the agents in another sub-network are governed by the double-integrator dynamics:

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t)-\Lambda_2v_i(t),\end{cases}\qquad\forall i\in I_2=\{n+1,\cdots,n+m\},\tag{4}$$

where $x_i(t),v_i(t)\in R^p$, and the matrix $-\Lambda_2\in R^{p\times p}$ is Hurwitz. Here the term $-\Lambda_2v_i$ represents the velocity damping term [29]. It should be especially noted that in [29], the damping force is in proportion to the magnitude of velocity, which is not required here and thus make our results more general. Besides, $u_i(t)\in R^p$ is the control input or the group consensus protocol.

The following lemmas will be used to derive the main results of this paper.

Lemma 2. *([36]). Let $\phi:R\to R$ be a uniformly continuous function on $[0,\infty)$. Suppose that $\lim_{t\to\infty}\int_0^t\phi(\tau)d\tau$ exists and is finite, then $\phi(t)\to0$ as $t\to\infty$.*

Lemma 3. *([37]). Given the system*

$$\dot{x} = Ax + u, \tag{5}$$

suppose that the system $\dot{x} = Ax$ is asymptotically stable and $u \in L_\infty$. Then the solution $x(t)$ of system (5) and its derivative $\dot{x}(t)$ belong to L_∞. In addition, $x(t)$ is uniformly continuous, where L_∞ is defined as the set $\{f : R_{\geq 0} \to R^n \mid \|f\|_\infty < \infty\}$ with $\|f\|_\infty = \sup_{t \geq 0} \|f(t)\|$.

3. Main results

In this section, the group consensus problem of the heterogeneous multi-agent systems (2) and (4) is investigated. A novel group consensus protocol and a time-varying estimator of the uncertain parameters θ_i, $i \in I_1$ are successfully applied to the analysis of consensus. Furthermore, the group consensus with switching topology is also considered.

In the heterogeneous multi-agent systems, time delay of information exchange between agents of different sub-networks is usually unavoidable with practical reasons such as the communication congestion of the channels, the spreading speed of the hardware implementation, and thus is considered in our framework, as shown in Fig. 1. Furthermore, let a network topology (G, x) consist of $n+m$ $(n, m > 1)$ agents. The first n agents constitute a sub-network G_1 with the Laplacian matrix L_1, and the rest m agents constitute the other sub-network G_2 with the Laplacian matrix L_2. Meanwhile, it is assumed that the sub-networks G_1 and G_2 are both strongly connected, and the information exchange between G_1 and G_2 is undirected. Such assumption indicates that the information may exchange more frequently between homogeneous agents than between heterogeneous agents, which conforms to the law of nature [27].

Denote $I_1 = \{1, 2, \ldots n\}$, $I_2 = \{n+1, n+2, \ldots n+m\}$, $V_1 = \{v_1, v_2, \ldots v_n\}$, $V_2 = \{v_{n+1}, \ldots v_{n+m}\}$, $N_{1i} = \{v_j \in V_1 \mid (v_j, v_i) \in \xi\}$, $N_{2i} = \{v_j \in V_2 \mid (v_j, v_i) \in \xi\}$, and denote by $S_T = \{T_{ij} \mid T_{ij} > 0, \forall i \in I_1, \ j \in I_2$ or $\forall i \in I_2, \ j \in I_1\}$ the set of time delay. The following definitions are needed in order to obtain the main results.

Definition 1. The protocols τ and u are considered to asymptotically solve the group consensus problem, if for any initial states of systems (2) and (4), the states of agents satisfy $\lim_{t\to\infty} \|q_i - q_j\| = \lim_{t\to\infty} \|\dot{q}_i - \dot{q}_j\| = 0$, $\forall i, j \in I_1$, $\lim_{t\to\infty} \|x_i - x_j\| = \lim_{t\to\infty} \|v_i - v_j\| = 0$, $\forall i, j \in I_2$. Furthermore, the protocols τ and u are considered to asymptotically solve the time-delay group consensus problem, if for any $i \in I_1$, $j \in I_2$, and $T \in S_T$, the states of agents satisfy one of the following conditions:

$$\lim_{t\to\infty} \|q_i(t-T) - x_j(t)\| = \lim_{t\to\infty} \|\dot{q}_i(t-T) - v_j(t)\| = 0,$$

$$\lim_{t\to\infty} \|q_i(t) - x_j(t-T)\| = \lim_{t\to\infty} \|\dot{q}_i(t) - v_j(t-T)\| = 0.$$

To solve the group consensus problem, a novel group consensus protocol is defined as follows:

$$\tau_i = Y_i(q_i, \dot{q}_i, -\Lambda_1 \dot{q}_i, -\Lambda_1 q_i)\hat{\theta}_i + \omega_i^{(1)} \Lambda_1 \sum_{j \in N_{1i}} a_{ij}(q_j - q_i) + \omega_i^{(1)} \sum_{j \in N_{1i}} a_{ij}(\dot{q}_j - \dot{q}_i)$$
$$+ \sum_{j \in N_{2i}} a_{ij}[\Lambda_2 x_j(t-T_{ij}) + \dot{x}_j(t-T_{ij}) - \Lambda_1 q_i(t) - \dot{q}_i(t)], \quad \forall i \in I_1, \tag{6}$$

$$u_i = \omega_i^{(2)} \Lambda_2 \sum_{j \in N_{2i}} a_{ij}(x_j - x_i) + \omega_i^{(2)} \sum_{j \in N_{2i}} a_{ij}(v_j - v_i)$$
$$+ \sum_{j \in N_{1i}} a_{ij}[\Lambda_1 q_j(t-T_{ij}) + \dot{q}_j(t-T_{ij}) - \Lambda_2 x_i(t) - v_i(t)], \quad \forall i \in I_2, \tag{7}$$

where $\Lambda_1, \Lambda_2 \in R^{p\times p}$, and $-\Lambda_1$, $-\Lambda_2$ are both Hurwitz, $\hat{\theta}_i$ is the estimation of the unknown parameter θ_i, the vectors $\omega^{(1)} = [\omega_1^{(1)}, \cdots, \omega_n^{(1)}]^T$ and $\omega^{(2)} = [\omega_{n+1}^{(2)}, \cdots, \omega_{n+m}^{(2)}]^T$ satisfy $L_1^T \omega^{(1)} = 0$ and $L_2^T \omega^{(2)} = 0$, respectively, and the matrix $Y_i(q_i, \dot{q}_i, -\Lambda_1 \dot{q}_i, -\Lambda_1 q_i)$ is defined in Eq. (3). According to (6) and (7), time delay only affects the information exchange between different sub-networks, and it is not assumed that $T_{ij} = T_{ji}$, $\forall i \in I_1, j \in I_2$.

Remark 1. The matrices Λ_1 and Λ_2 are called the inner coupling matrices [38] describing the interactions between different state components of agents, and are always assumed to be symmetric or diagonal in the most of the former works. However, in this paper, it is only assumed that $-\Lambda_1$ and $-\Lambda_2$ are both Hurwitz. Moreover, the protocol (6) includes the term $Y_i(q_i, \dot{q}_i, -\Lambda \dot{q}_i, -\Lambda q_i)\hat{\theta}_i$, which provides an effective mechanism to cope with uncertainties. In contrast to the protocol in [39], the protocol (6) does not require the existence of the damping term in the Euler–Lagrange system.

Then, the main result of the paper is given by the following theorem:

Theorem 1. *Consider a network with n Euler–Lagrange agents and m double-integrator agents governed by the form (2) and (4),*

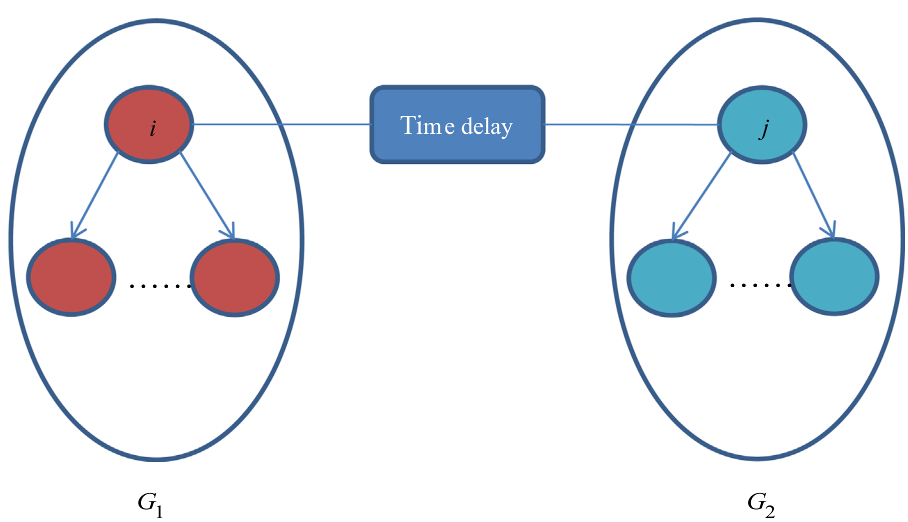

Fig. 1. The structure of the heterogeneous multi-agent systems.

respectively, let the protocols be defined by (6) and (7), together with the estimation law

$$\dot{\hat{\theta}}_i = -\Gamma_i^{-1} Y_i^T(q_i, \dot{q}_i, -\Lambda_1\dot{q}_i, -\Lambda_1 q_i)[\dot{q}_i + \Lambda_1 q_i], \quad \forall i \in I_1, \tag{8}$$

where $\Gamma_i \in R^{p\times p}$ is a positive definite matrix, then the protocols (6) and (7) solve the group consensus problem. Furthermore, if the matrices Λ_1, Λ_2 satisfy $\Lambda_1 = \Lambda_2$, then the protocols (6) and (7) solve the time-delay group consensus problem.

Proof. Firstly, denote the auxiliary variables by $s_i = \dot{q}_i + \Lambda_1 q_i$, $\forall i \in I_1$, $r_j = v_j + \Lambda_2 x_j$, $\forall j \in I_2$, and $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. According to (P3), it follows that:

$$M_i(q_i)[-\Lambda_1\dot{q}_i] + C_i(q_i,\dot{q}_i)[-\Lambda_1 q_i] + g_i(q_i) = Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i, \tag{9}$$

Hence, from Eqs. (2) and (7), it implies

$$M_i(q_i)[\ddot{q}_i + \Lambda_1\dot{q}_i] + C_i(q_i,\dot{q}_i)[\dot{q}_i + \Lambda_1 q_i] = \tau_i - Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i, \tag{10}$$

i.e.,

$$M_i(q_i)\dot{s}_i + C_i(q_i,\dot{q}_i)s_i = \tau_i - Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i, \tag{11}$$

Next, we consider the following auxiliary function

$$\begin{aligned} V(t) &= \frac{1}{2}\sum_{i=1}^{n}\left[s_i^T M_i(q_i)s_i + \tilde{\theta}_i^T\Gamma_i\tilde{\theta}_i\right] + \frac{1}{2}\sum_{j=n+1}^{n+m} r_j^T r_j \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}\int_{t-T_{ji}}^{t} s_i^T(\tau)s_i(\tau)d\tau \\ &\quad + \frac{1}{2}\sum_{j=n+1}^{n+m}\sum_{i=1}^{n} a_{ji}\int_{t-T_{ij}}^{t} r_j^T(\tau)r_j(\tau)d\tau, \end{aligned} \tag{12}$$

and we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{n}\left[s_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T\dot{M}_i(q_i)s_i + \tilde{\theta}_i^T\Gamma_i\dot{\tilde{\theta}}_i\right] + \sum_{i=n+1}^{n+m} r_j^T\dot{r}_j \\ &\quad + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\frac{a_{ij}}{2}[s_i^T(t)s_i(t) - s_i^T(t-T_{ji})s_i(t-T_{ji})] \\ &\quad + \sum_{j=n+1}^{n+m}\sum_{i=1}^{n}\frac{a_{ji}}{2}[r_j^T(t)r_j(t) - r_j^T(t-T_{ij})r_j(t-T_{ij})]. \end{aligned} \tag{13}$$

Considering the term $s_i^T M_i(q_i)\dot{s}_i + (1/2)s_i^T\dot{M}_i(q_i)s_i + \tilde{\theta}_i^T\Gamma_i\dot{\tilde{\theta}}_i$, according to (P2), Eqs. (8) and (11), we have

$$\begin{aligned} & s_i^T M_i(q_i)\dot{s}_i + \frac{1}{2}s_i^T\dot{M}_i(q_i)s_i + \tilde{\theta}_i^T\Gamma_i\dot{\tilde{\theta}}_i \\ &= s_i^T\left[\tau_i - Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i - C_i(q_i,\dot{q}_i)s_i\right] + \frac{1}{2}s_i^T\dot{M}_i(q_i)s_i \\ &\quad - \tilde{\theta}_i^T Y_i^T(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)s_i \\ &= s_i^T\left[\tau_i - Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i\right] - s_i^T C_i(q_i,\dot{q}_i)s_i + \frac{1}{2}s_i^T\dot{M}_i(q_i)s_i \\ &\quad - \tilde{\theta}_i^T Y_i^T(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)s_i \\ &= s_i^T\left[\tau_i - Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)\theta_i\right] - \tilde{\theta}_i^T Y_i^T(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1 q_i)s_i \\ &= s_i^T\left[\omega_i^{(1)}\sum_{j\in N_{1i}} a_{ij}(s_j - s_i)\right] + s_i^T\left[\sum_{j\in N_{2i}} a_{ij}(r_j(t-T_{ij}) - s_i(t))\right]. \end{aligned} \tag{14}$$

Then, according to Eqs. (4) and (7), it follows that:

$$\begin{aligned} r_j^T\dot{r}_j &= r_j^T[\dot{v}_j + \Lambda_2\dot{x}_j] \\ &= r_j^T[u_j - \Lambda_2 v_j + \Lambda_2 v_j] \\ &= r_j^T\left[\omega_j^{(2)}\sum_{i\in N_{2j}} a_{ji}(r_i - r_j)\right] + r_j^T\left[\sum_{i\in N_{1j}} a_{ji}(s_i(t-T_{ji}) - r_j(t))\right]. \end{aligned} \tag{15}$$

Substituting Eqs. (13) and (14) into Eq. (15), we obtain

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{n} s_i^T\left[\omega_i^{(1)}\sum_{j\in N_{1i}} a_{ij}(s_j - s_i)\right] + \sum_{i=1}^{n} s_i^T\left[\sum_{j\in N_{2i}} a_{ij}(r_j(t-T_{ij}) - s_i(t))\right] \\ &\quad + \sum_{j=n+1}^{n+m} r_j^T\left[\omega_j^{(2)}\sum_{i\in N_{2j}} a_{ji}(r_i - r_j)\right] + \sum_{j=n+1}^{n+m} r_j^T\left[\sum_{i\in N_{1j}} a_{ji}(s_i(t-T_{ji}) - r_j(t))\right] \\ &\quad + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\frac{a_{ij}}{2}[s_i^T(t)s_i(t) - s_i^T(t-T_{ji})s_i(t-T_{ji})] \\ &\quad + \sum_{j=n+1}^{n+m}\sum_{i=1}^{n}\frac{a_{ji}}{2}\left[r_j^T(t)r_j(t) - r_j^T(t-T_{ij})r_j(t-T_{ij})\right] \\ &= \sum_{i=1}^{n} s_i^T\left[\omega_i^{(1)}\sum_{j\in N_{1i}} a_{ij}(s_j - s_i)\right] + \sum_{j=n+1}^{n+m} r_j^T\left[\omega_j^{(2)}\sum_{i\in N_{2j}} a_{ji}(r_i - r_j)\right] \\ &\quad + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}s_i^T(r_j(t-T_{ij}) - s_i(t)) + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}r_j^T(s_i(t-T_{ji}) - r_j(t)) \\ &\quad + \sum_{i=1}^{n}\sum_{j=n+1}^{n+m}\frac{a_{ij}}{2}[s_i^T(t)s_i(t) - s_i^T(t-T_{ji})s_i(t-T_{ji})] \\ &\quad + \sum_{j=n+1}^{n+m}\sum_{i=1}^{n}\frac{a_{ij}}{2}\left[r_j^T(t)r_j(t) - r_j^T(t-T_{ij})r_j(t-T_{ij})\right] \\ &= \sum_{i=1}^{n} s_i^T\left[\omega_i^{(1)}\sum_{j\in N_{1i}} a_{ij}(s_j - s_i)\right] + \sum_{j=n+1}^{n+m} r_j^T\left[\omega_j^{(2)}\sum_{i\in N_{2j}} a_{ji}(r_i - r_j)\right] \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t) - r_j(t-T_{ij})]^T[s_i(t) - r_j(t-T_{ij})] \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t-T_{ji}) - r_j(t)]^T[s_i(t-T_{ji}) - r_j(t)]. \end{aligned} \tag{16}$$

By using vector notations, we can rewrite Eq. (16) with a more compact form as follows:

$$\begin{aligned} \dot{V}(t) &= -[s_1^T,\ldots,s_n^T]\{diag(\omega^{(1)})L_1 \otimes I_p\}\begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix} \\ &\quad - [r_{n+1}^T,\ldots,r_{n+m}^T]\{diag(\omega^{(2)})L_2 \otimes I_p\}\begin{bmatrix} r_{n+1} \\ \vdots \\ r_{n+m} \end{bmatrix} \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t) - r_j(t-T_{ij})]^T[s_i(t) - r_j(t-T_{ij})] \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t-T_{ji}) - r_j(t)]^T[s_i(t-T_{ji}) - r_j(t)] \\ &= -[s_1^T,\ldots,s_n^T]\left\{\left[\frac{diag(\omega^{(1)})L_1 + L_1^T diag(\omega^{(1)})}{2}\right] \otimes I_p\right\}\begin{bmatrix} s_1 \\ \vdots \\ s_n \end{bmatrix} \\ &\quad - [r_{n+1}^T,\ldots,r_{n+m}^T]\left\{\left[\frac{diag(\omega^{(2)})L_2 + L_2^T diag(\omega^{(2)})}{2}\right] \otimes I_p\right\}\begin{bmatrix} r_{n+1} \\ \vdots \\ r_{n+m} \end{bmatrix} \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t) - r_j(t-T_{ij})]^T[s_i(t) - r_j(t-T_{ij})] \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t-T_{ji}) - r_j(t)]^T[s_i(t-T_{ji}) - r_j(t)], \end{aligned} \tag{17}$$

where I_p denotes the $p \times p$ identity matrix. According to Lemma 1, it follows $\dot{V}(t) \le 0$, indicating that $V(t)$ is not increasing with t. Since $V(t) \ge 0$, $\forall t \in [0, +\infty)$, the function $V(t)$ is bounded, and this yields that $S \in L_\infty$, $\tilde{\theta} \in L_\infty$, and $R \in L_\infty$, where $S = [s_1^T, \ldots, s_n^T]^T$, $\tilde{\theta} = [\tilde{\theta}_1^T, \ldots, \tilde{\theta}_{n+m}^T]^T$, and $R = [r_{n+1}^T, \ldots, r_{n+m}^T]^T$. By the definition of s_i, r_j, and Lemma 3, we can get that $q_i(t)$, $\dot{q}_i(t) \in L_\infty$, and $x_i(t)$, $v_i(t) \in L_\infty$, then, $\Lambda_1 q_i(t)$, $\Lambda_1\dot{q}_i(t)$ and $\Lambda_2 v_i(t)$ are also bounded. According to (P1), the matrix $C_i(q_i, \dot{q}_i)$ is bounded, which implies that the regression matrix $Y_i(q_i, \dot{q}_i, -\Lambda_1\dot{q}_i, -\Lambda_1 q_i)$ is also bounded from Eq. (9). Hence, according to Eq. (6) and (P1), we obtain that $\dot{S}$ is bounded. Since $\dot{r}_i(t) = \ddot{x}_i(t) + \Lambda_2\dot{x}_i(t) = \dot{v}_i(t) + \Lambda_2 v_i(t) = u_i(t)$, according to Eq. (7), $\dot{R}$ is also bounded. Therefore, we have that $\ddot{V}(t)$ is a bounded function and $\dot{V}(t)$ is a uniformly continuous function on $[0,\infty)$. Since Lemma 2 indicates $\lim_{t\to\infty}\dot{V}(t)=0$, due to the assumption that sub-networks G_1 and G_2 are strongly connected, we can conclude that

$$\lim_{t\to\infty}\|s_i-s_j\|=0,\ \forall i,j\in I_1, \tag{18}$$

and

$$\lim_{t\to\infty}\|r_i-r_j\|=0,\ \forall i,j\in I_2. \tag{19}$$

From the definition of s_i and r_i, we have

$$s_i-s_j=\dot{q}_i-\dot{q}_j+\Lambda_1(q_i-q_j)$$
$$r_i-r_j=v_i-v_j+\Lambda_2(x_i-x_j). \tag{20}$$

Because the matrices $-\Lambda_1$ and $-\Lambda_2$ are Hurwitz, we obtain that

$$\lim_{t\to\infty}\|q_i-q_j\|=\lim_{t\to\infty}\|\dot{q}_i-\dot{q}_j\|=0,\quad \forall i,j\in I_1,$$
$$\lim_{t\to\infty}\|x_i-x_j\|=\lim_{t\to\infty}\|v_i-v_j\|=0,\quad \forall i,j\in I_2. \tag{21}$$

Thus the protocols (6) and (7) solve the group consensus problem of the heterogeneous multi-agent systems. Furthermore, if $\Lambda_1=\Lambda_2$, we have

$$\begin{aligned}s_i(t)-r_j(t-T_{ij})&=\dot{q}_i(t)+\Lambda_1q_i(t)-\dot{x}_j(t-T_{ij})-\Lambda_1x_j(t-T_{ij})\\&=(\dot{q}_i(t)-\dot{x}_j(t-T_{ij}))+\Lambda_1(q_i(t)-x_j(t-T_{ij})),\end{aligned} \tag{22}$$

and

$$\begin{aligned}s_i(t-T_{ji})-r_j(t)&=\dot{q}_i(t-T_{ji})+\Lambda_1q_i(t-T_{ji})-\dot{x}_j(t)-\Lambda_1x_j(t)\\&=(\dot{q}_i(t-T_{ji})-\dot{x}_j(t))+\Lambda_1(q_i(t-T_{ji})-x_j(t)).\end{aligned} \tag{23}$$

Hence, according to Eq. (17), we can conclude that if there exists the information exchange between agents i and j in the different sub-networks,

$$\lim_{t\to\infty}\|q_i(t)-x_j(t-T_{ij})\|=\lim_{t\to\infty}\|\dot{q}_i(t)-v_j(t-T_{ij})\|=0,$$
$$\lim_{t\to\infty}\|q_i(t-T_{ji})-x_j(t)\|=\lim_{t\to\infty}\|\dot{q}_i(t-T_{ji})-v_j(t)\|=0. \tag{24}$$

Moreover, for any $T\in S_T$, there exist indexes $i_0\in I_1$ and $j_0\in I_2$ such that $T=T_{i_0j_0}$ or $T=T_{j_0i_0}$. Without loss of generality, it is assumed that $T=T_{i_0j_0}$, and it follows Eq. (24) that

$$\lim_{t\to\infty}\|q_{i_0}(t)-x_{j_0}(t-T_{i_0j_0})\|=\lim_{t\to\infty}\|\dot{q}_{i_0}(t)-v_{j_0}(t-T_{i_0j_0})\|=0. \tag{25}$$

Therefore, for any $i\in I_1$ and $j\in I_2$, we have

$$\begin{aligned}\|q_i(t)-x_j(t-T)\|&=\|q_i(t)-q_{i_0}(t)+q_{i_0}(t)-x_{j_0}(t-T_{i_0j_0})\\&\quad+x_{j_0}(t-T_{i_0j_0})-x_j(t-T_{i_0j_0})\|\\&\le\|q_i(t)-q_{i_0}(t)\|+\|q_{i_0}(t)-x_{j_0}(t-T_{i_0j_0})\|\\&\quad+\|x_{j_0}(t-T_{i_0j_0})-x_j(t-T_{i_0j_0})\|.\end{aligned} \tag{26}$$

then

$$\lim_{t\to\infty}\|q_i(t)-x_j(t-T)\|=0, \tag{27}$$

Thus the protocols (6) and (7) solve the time-delay group consensus problem of the heterogeneous multi-agent systems. This completes the proof.

Remark 2. In Theorem 1, by the protocols (6) and (7), we solve the group consensus problem asymptotically. Note that the protocols in [23,24] cannot be directly applied to the heterogeneous system, since they only consider the single-integrator dynamics in their models. Furthermore, we give the definition of the time-delay group consensus, and it is proved that the protocols (6) and (7) can solve the time-delay group consensus problem provided that the inner coupling matrices are equal in the different sub-networks.

In practice, the relationships between homogeneous agents may vary over time [27], and their interconnection topology may also be dynamically changing. Consequently, switching topologies between homogeneous agents should be taken into account in some cases. Suppose that there are N topologies $G^1,\dots,G^N$, and for each complex network (G^ρ,V), its sub-networks G_1^ρ and G_2^ρ are strongly connected. Then, a switching topology is defined by a switching signal $\sigma(t):[0,+\infty)\to\mathfrak{I}$, which is represented by a piecewise constant function $\{\sigma_k\}:(t_{k-1},t_k]\to\sigma_k$ with the time sequence $\{t_k\}$ satisfying: $0=t_0<t_1<t_2<\dots<t_k<\dots,\quad \lim_{k\to\infty}t_k=\infty$, and there is a constant μ (called dwell time) with $t_{k+1}-t_k\ge\mu$ for all $k\ge0$, which helps to avoid the case of infinitely fast switching (chattering). Moreover, $\sigma(t)=\rho$ means that the topology G^ρ is adopted at time t. Note that the interconnection topology between heterogeneous agents does not change in this process.

Then, a group consensus protocol with switching topologies between homogeneous agents is defined as follows: $\forall t\in(t_{k-1},t_k]$,

$$\begin{aligned}\tau_i&=Y_i(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1q_i)\hat{\theta}_i+\omega_i^{\sigma(t)(1)}\Lambda_1\sum_{j\in N_{1i}}a_{ij}^{\sigma(t)}(q_j-q_i)\\&\quad+\omega_i^{\sigma(t)(1)}\sum_{j\in N_{1i}}a_{ij}^{\sigma(t)}(\dot{q}_j-\dot{q}_i)\\&\quad+\sum_{j\in N_{2i}}a_{ij}[\Lambda_2x_j(t-T_{ij})+\dot{x}_j(t-T_{ij})-\Lambda_1q_i(t)-\dot{q}_i(t)],\quad \forall i\in I_1,\end{aligned} \tag{28}$$

$$\begin{aligned}u_i&=\omega_i^{\sigma(t)(2)}\Lambda_2\sum_{j\in N_{2i}}a_{ij}^{\sigma(t)}(x_j-x_i)+\omega_i^{\sigma(t)(2)}\sum_{j\in N_{2i}}a_{ij}^{\sigma(t)}(v_j-v_i)\\&\quad+\sum_{j\in N_{1i}}a_{ij}[\Lambda_1q_j(t-T_{ij})+\dot{q}_j(t-T_{ij})-\Lambda_2x_i(t)-v_i(t)],\quad \forall i\in I_2,\end{aligned} \tag{29}$$

where the vectors $\omega_{\sigma(t)}^{(1)}=[\omega_{1\sigma(t)}^{(1)},\dots,\omega_{n\sigma(t)}^{(1)}]^T$ and $\omega_{\sigma(t)}^{(2)}=[\omega_{n+1\sigma(t)}^{(2)},\dots,\omega_{n+m\sigma(t)}^{(2)}]^T$ satisfy $L_{\sigma(t)1}^T\omega_{\sigma(t)}^{(1)}=0$ and $L_{\sigma(t)2}^T\omega_{\sigma(t)}^{(2)}=0$, respectively.

Remark 2. Due to the switching topologies, Lemma 2 cannot be directly used to establish group consensus. In fact, the function $\dot{V}(t)$ in Theorem 1 is piecewise continuous, which does not satisfy the condition of Lemma 2. In this paper, the Barbalat-like lemma originally established in [40] is adopted to solve the group consensus problem with switching topologies.

The following Theorem shows that the group consensus problem could be solved even when the communication graph is dynamic.

Theorem 2. *Consider that the complex network (G,V) has switching topologies between homogeneous agents, whose dynamics are governed by either Eqs. (2) or (4). Provided by the estimation law*

$$\dot{\hat{\theta}}_i=-\Gamma_i^{-1}Y_i^T(q_i,\dot{q}_i,-\Lambda_1\dot{q}_i,-\Lambda_1q_i)[\dot{q}_i+\Lambda_1q_i],\quad i\in I_1, \tag{30}$$

where $\Gamma_i\in R^{p\times p}$ is a positive definite matrix, the protocols (28) and (29) solve the group consensus problem. Furthermore, if the matrices $\Lambda_1,\ \Lambda_2$ satisfy $\Lambda_1=\Lambda_2$, then the protocols (28) and (29) solve the time-delay group consensus problem.

Proof. Denote the auxiliary variables by $s_i=\dot{q}_i+\Lambda_1q_i,\ \forall i\in I_1$, $r_j=v_j+\Lambda_2x_j,\ \forall j\in I_2$, and $\tilde{\theta}_i=\hat{\theta}_i-\theta_i$. Consider the following auxiliary function

$$\begin{aligned}V(t)&=\frac{1}{2}\sum_{i=1}^{n}[s_i^TM_i(q_i)s_i+\tilde{\theta}_i^T\Gamma_i\tilde{\theta}_i]+\frac{1}{2}\sum_{j=n+1}^{n+m}r_j^Tr_j+\frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m}a_{ij}\\&\quad\int_{t-T_{ji}}^{t}s_i^T(\tau)s_i(\tau)d\tau+\frac{1}{2}\sum_{j=n+1}^{n+m}\sum_{i=1}^{n}a_{ji}\int_{t-T_{ij}}^{t}r_j^T(\tau)r_j(\tau)d\tau,\end{aligned} \tag{31}$$

Then, $\forall t\in(t_{k-1},t_k]$, the derivative of $V(t)$ along the trajectory of Eqs. (2) and (4) is given by

$$\begin{aligned}\dot{V}(t)&=-[s_1^T,\dots,s_n^T]\left\{\left[\frac{diag(\omega_{\sigma(t)}^{(1)})L_{1\sigma(t)}+L_{1\sigma(t)}^Tdiag(\omega_{\sigma(t)}^{(1)})}{2}\right]\otimes I_p\right\}\begin{bmatrix}s_1\\\vdots\\s_n\end{bmatrix}\\&\quad-[r_{n+1}^T,\dots,r_{n+m}^T]\left\{\left[\frac{diag(\omega_{\sigma(t)}^{(2)})L_{2\sigma(t)}+L_{2\sigma(t)}^Tdiag(\omega_{\sigma(t)}^{(2)})}{2}\right]\otimes I_p\right\}\end{aligned}$$

$$\begin{bmatrix} r_{n+1} \\ \vdots \\ r_{n+m} \end{bmatrix} - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t)-r_j(t-T_{ij})]^T[s_i(t)-r_j(t-T_{ij})]$$
$$-\frac{1}{2}\sum_{i=1}^{n}\sum_{j=n+1}^{n+m} a_{ij}[s_i(t-T_{ji})-r_j(t)]^T[s_i(t-T_{ji})-r_j(t)] \le 0, \tag{32}$$

Since $V(t)\ge 0$, $\forall t\in[0,\infty)$, it yields that $V(t)$ is bounded, which implies that $S\in L_\infty$, $R\in L_\infty$, and $\tilde{\theta}\in L_\infty$, with $S=[s_1^T,\dots,s_n^T]^T$, $R=[r_{n+1}^T,\dots,r_{n+m}^T]^T$, and $\tilde{\theta}=[\tilde{\theta}_1^T,\cdots,\tilde{\theta}_{n+m}^T]^T$.

According to the assumption on the switching time sequence, there exists a subsequence of switching times $\{t_{w_k}\}$ such that the time intervals $t_{w_{k+1}}-t_{w_k}\ge\tau$, $k=1,2,\dots$, apparently, $\tau>\mu$, and $\sigma(t)=\rho$ on these time intervals. Denote the union of these time intervals by Ξ, and construct the auxiliary function as follows:

$$x_\Xi(t)=\begin{cases} -\dot{V}(t), & t\in\Xi \\ 0, & t\notin\Xi \end{cases}. \tag{33}$$

Obviously, $x_\Xi(t)$ is piecewise continuous, and $x_\Xi(t)\ge 0$. From Eq. (32), we obtain that $\forall t\ge 0$,

$$\int_0^t x_\Xi(s)ds \le V(t_{w_1})-V(t)\le V(t_{w_1}). \tag{34}$$

Thus, $\int_0^\infty x_\Xi(s)ds$ exists and is finite. Next, we proceed to prove that $\lim_{t\to\infty} x_\Xi(t)=0$. Suppose that it is not true, then there exists a constant $\varepsilon>0$ and an infinite sequence of times $\{t_k'\}\subset\Xi$ such that $x_\Xi(t_k')\ge\varepsilon$, $\forall k$. Due to the fact that $\ddot{V}(t)$ is bounded on the interval $(t_{w_k},t_{w_k+1}]$, the function $x_\Xi(t)$ is uniformly continuous on $(t_{w_k},t_{w_k+1}]$. Therefore, there exists a constant $\eta>0$ such that for any t belonging to time interval of length η, one has $x_\Xi(t)\ge(\varepsilon/2)$, which results in a contradiction. So, we have $\lim_{t\to\infty} x_\Xi(t)=0$, and $\lim_{t\to\infty}\dot{V}(t)=0$. Due to the assumption that sub-networks G_1^ρ and G_2^ρ are strongly connected, we can conclude that

$$\lim_{t\to\infty}\|s_i-s_j\|=0,\ \forall i,j\in I_1, \tag{35}$$

and

$$\lim_{t\to\infty}\|r_i-r_j\|=0,\ \forall i,j\in I_2. \tag{36}$$

Similar to Theorem 1, it follows that the protocols (28) and (29) solve the group consensus problem. Moreover, if $\Lambda_1=\Lambda_2$, the protocol (28) and (29) also solve the time-delay group consensus problem. This completes the proof.

4. Numerical simulation

In this section, two numerical examples are provided to illustrate the effectiveness of the proposed theoretical results.

In the simulation, the dynamics of Euler–Lagrange system is described as

$$\begin{bmatrix} M_{11}^i & M_{12}^i \\ M_{21}^i & M_{22}^i \end{bmatrix}\begin{bmatrix} \ddot{q}_i^{(1)} \\ \ddot{q}_i^{(2)} \end{bmatrix}+\begin{bmatrix} -h_i\dot{q}_i^{(2)} & -h_i(\dot{q}_i^{(1)}+\dot{q}_i^{(2)}) \\ h_i\dot{q}_i^{(1)} & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_i^{(1)} \\ \dot{q}_i^{(2)} \end{bmatrix}=\begin{bmatrix} \tau_i^{(1)} \\ \tau_i^{(2)} \end{bmatrix},$$
$$i\in I_1, \tag{37}$$

where $q_i=[q_i^{(1)},q_i^{(2)}]^T$, $\tau_i=[\tau_i^{(1)},\tau_i^{(2)}]^T$, and

$$M_{11}^i=a_1+2a_3\ \cos(q_i^{(2)})+2a_4\ \sin(q_i^{(2)})$$
$$M_{12}^i=M_{21}^i=a_2+a_3\ \cos(q_i^{(2)})+a_4\ \sin(q_i^{(2)})$$
$$M_{22}^i=a_2$$
$$h_i=a_3\ \sin(q_i^{(2)})-a_4\ \cos(q_i^{(2)}), \tag{38}$$

with

$$a_1=d_1+m_1l_1^2+d_2+m_2l_2^2+m_2l_3^2$$
$$a_2=d_2+m_2l_2^2$$
$$a_3=m_2l_3l_2\ \cos(\delta)$$
$$a_4=m_2l_3l_2\ \sin(\delta). \tag{39}$$

Here the parameters are chosen as $m_1=1$, $m_2=2$, $l_1=0.5$, $l_2=0.6$, $l_3=1$, $d_1=0.12$, $d_2=0.25$ and $\delta=(\pi/6)$. Moreover, according to Eq. (3), $Y_i(q_i,\dot{q}_i,x,y)\in R^{2\times 4}$ is defined as

$$Y_{11}^i=x_1,\quad Y_{12}^i=x_2,\quad Y_{21}^i=0,\quad Y_{22}^i=x_1+x_2,$$
$$Y_{13}^i=(2x_1+x_2)\cos(q_i^{(2)})-(\dot{q}_i^{(2)}y_1+\dot{q}_i^{(1)}y_2+\dot{q}_i^{(2)}y_2)\sin(q_i^{(2)})$$
$$Y_{14}^i=(2x_1+x_2)\sin(q_i^{(2)})+(\dot{q}_i^{(2)}y_1+\dot{q}_i^{(1)}y_2+\dot{q}_i^{(2)}y_2)\cos(q_i^{(2)})$$
$$Y_{23}^i=x_1\ \cos(q_i^{(2)})+\dot{q}_i^{(1)}y_1\ \sin(q_i^{(2)})$$
$$Y_{24}^i=x_1\ \sin(q_i^{(2)})-\dot{q}_i^{(1)}y_1\ \cos(q_i^{(2)}), \tag{40}$$

where $x=[x_1,x_2]^T$ and $y=[y_1,y_2]^T$.

Example 1. Consider ten agents in the heterogeneous multi-agent systems, with their interaction topology presented in Fig. 2.

From Fig. 2, the whole network G is divided into two sub-networks G_1 and G_2, and information exchanges exist between the

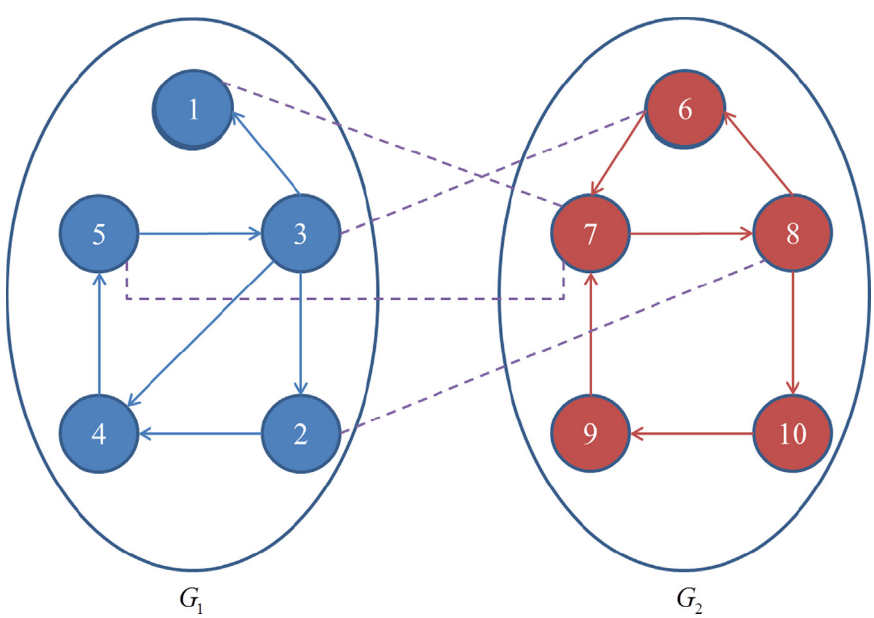

Fig. 2. The interaction topology of ten agents.

two sub-networks. It should be noted that the agents in G_1 are governed by the Euler–Lagrange system, and the agents in G_2 are described by the double-integrator dynamics. Furthermore, the Laplacian matrixes of G_1 and G_2 can be easily obtained as follows:

$$L_1=\begin{bmatrix} 0.2 & 0 & -0.2 & 0 & 0 \\ -0.3 & 0 & 0.3 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & -0.4 \\ 0 & -0.5 & -0.8 & 1.3 & 0 \\ 0 & 0 & 0 & -0.7 & 0.7 \end{bmatrix},$$
$$L_2=\begin{bmatrix} 0.6 & 0 & -0.6 & 0 & 0 \\ -0.9 & 1.4 & 0 & -0.5 & 0 \\ 0 & -0.3 & 0.3 & 0 & 0 \\ 0 & 0 & 0 & 0.4 & -0.4 \\ 0 & 0 & -0.5 & 0 & 0.5 \end{bmatrix}. \tag{41}$$

Then the adjacency matrix of information exchanges between G_1 and G_2 is given by

$$B=\begin{bmatrix} 0 & 0.3 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0.7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 \end{bmatrix}. \tag{42}$$

Note that the element 0.3 in B represents the weight between agent 1 and agent 7. According to Eq. (41), sub-networks G_1 and G_2 are strongly connected. Next, the initial values of q_i, $\dot{q}_i$, x_i and v_i are chosen randomly within $[-0.5, 0.5]\times[-0.5, 0.5]$. For simplicity, we choose the identical inner coupling matrices for the sub-networks G_1 and G_2, i.e.,

$$\Lambda_1=\Lambda_2=\begin{bmatrix} 1.5819 & -1.2273 \\ -0.6546 & 1.4181 \end{bmatrix}. \tag{43}$$

Meanwhile, the time-delays between different sub-networks are also identical, i.e., $T_{ij}=2$. According to Eqs. (2), (4), (6) and (7), the evolutions of q_i and $\dot{q}_i$ are shown in Fig. 3 and 4, while the evolutions of x_i and v_i are shown in Fig. 5 and 6, respectively. Apparently, the protocols (6) and (7) solve the group consensus problem. Moreover, the error between agents 1 and 6 is defined as follows:

$$e_1(t)=\sqrt{\sum_{i=1}^{2}[q_1^{(i)}(t-2)-x_6^{(i)}(t)]^2},$$
$$e_2(t)=\sqrt{\sum_{i=1}^{2}[\dot{q}_1^{(i)}(t-2)-v_6^{(i)}(t)]^2}, \tag{44}$$

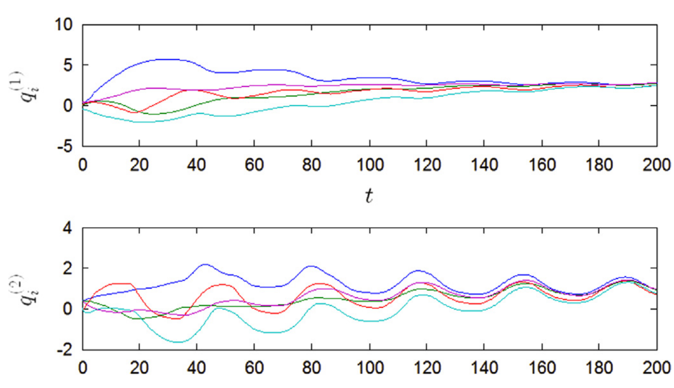

Fig. 3. The evolution of $q_i(t)$ with fixed topology.

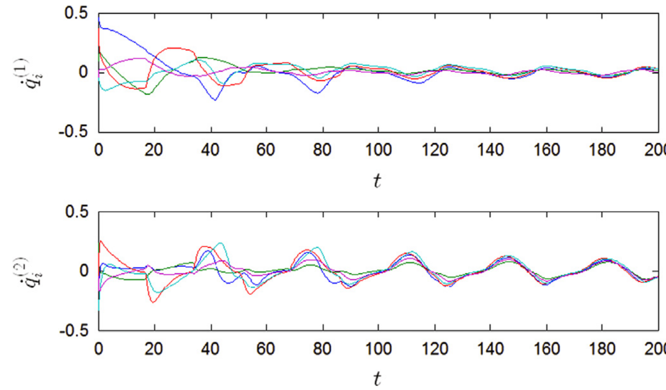

Fig. 4. The evolution of $\dot{q}_i(t)$ with fixed topology.

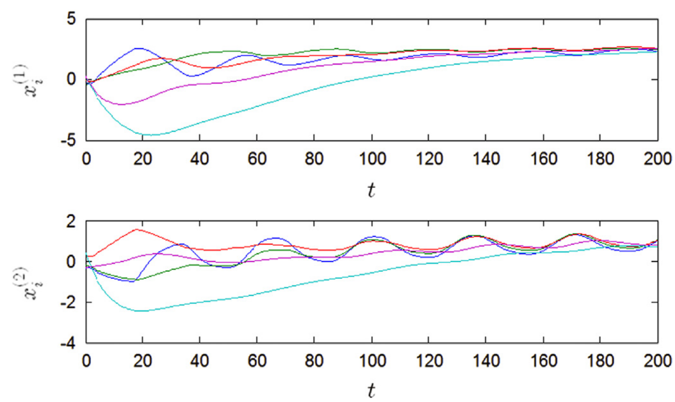

Fig. 5. The evolution of $x_i(t)$ with fixed topology.

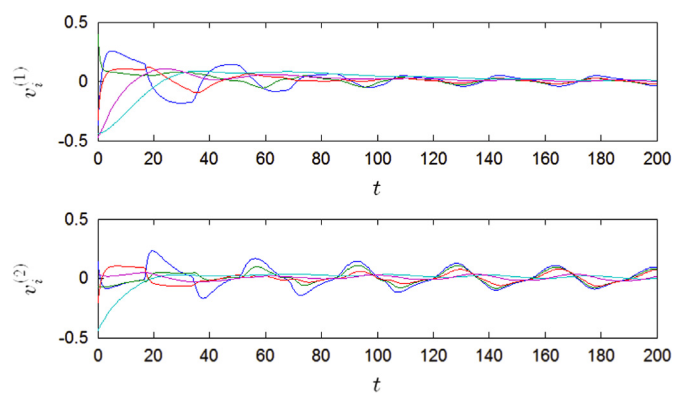

Fig. 6. The evolution of $v_i(t)$ with fixed topology.

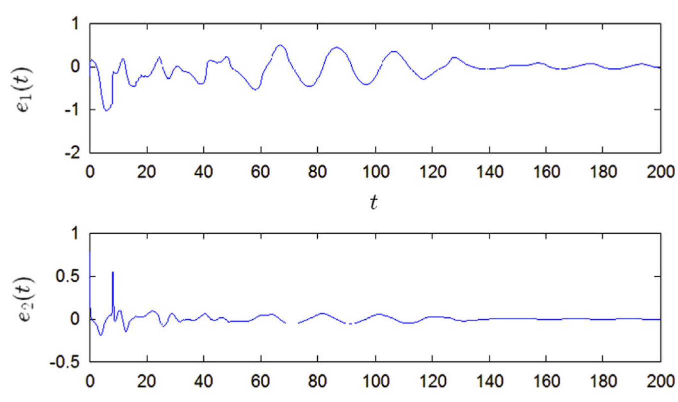

Fig. 7. The evolution of $e(t)$ with fixed topology.

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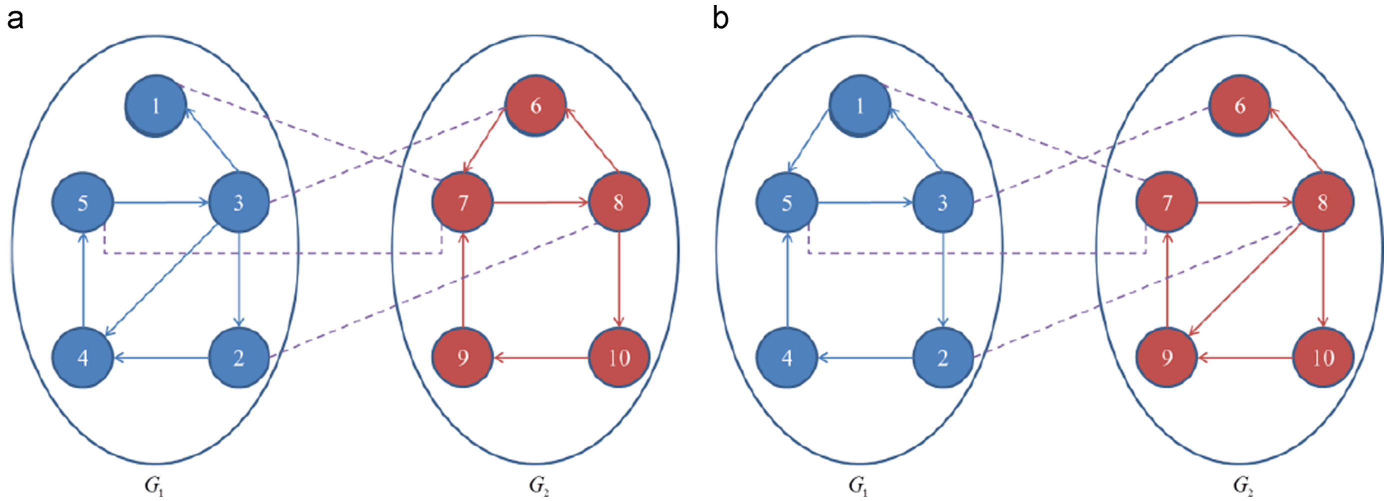

Fig. 8. Two different interaction topologies of ten agents.

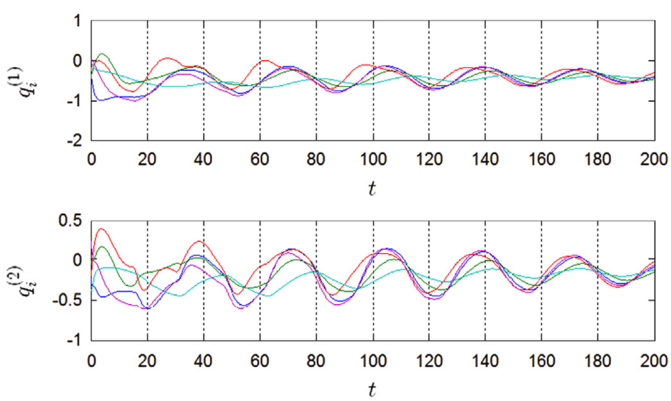

Fig. 9. The evolution of $q_i(t)$ with switching topologies.

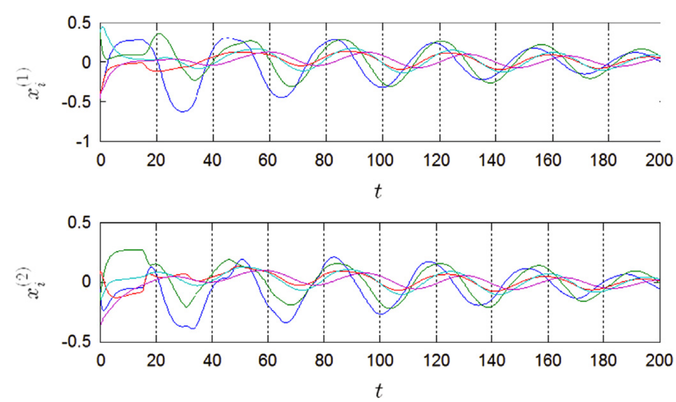

Fig. 11. The evolution of $x_i(t)$ with switching topologies.

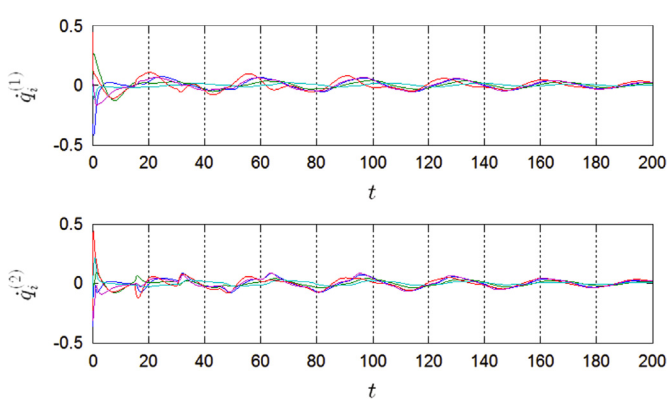

Fig. 10. The evolution of $\dot{q}_i(t)$ with switching topologies.

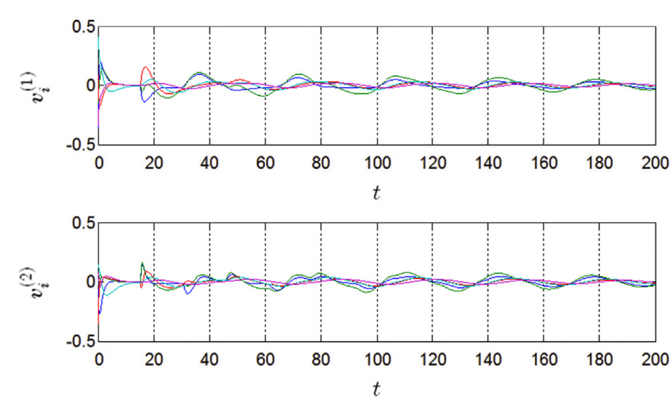

Fig. 12. The evolution of $v_i(t)$ with switching topologies.

where $x_6=[x_6^{(1)},x_6^{(2)}]^T$, $v_6=[v_6^{(1)},v_6^{(2)}]^T$. From Theorem 1, the protocols (6) and (7) could solve the time-delay group consensus problem provided that $\Lambda_1=\Lambda_2$, which is shown in Fig. 7.

Example 2. Consider ten agents with switching topologies shown in Fig. 8. Here, the dynamics of each agent in G_1 is also described by Euler–Lagrange systems (37), and the initial conditions and other parameters are all set the same as those in Example 1.

In this case, there are two different interaction topologies. For (a), the Laplacian matrixes and the adjacency matrix B are chosen the same as those in Example 1, while for (b), the Laplacian matrixes of G_1 and G_2 are as follows:

$$L_1=\begin{bmatrix} 0.5 & 0 & -0.5 & 0 & 0 \\ -0.8 & 1.5 & 0 & -0.7 & 0 \\ 0 & -0.8 & 0.8 & 0 & 0 \\ 0 & 0 & 0 & 0.7 & -0.7 \\ 0 & 0 & -0.3 & 0 & 0.3 \end{bmatrix},$$

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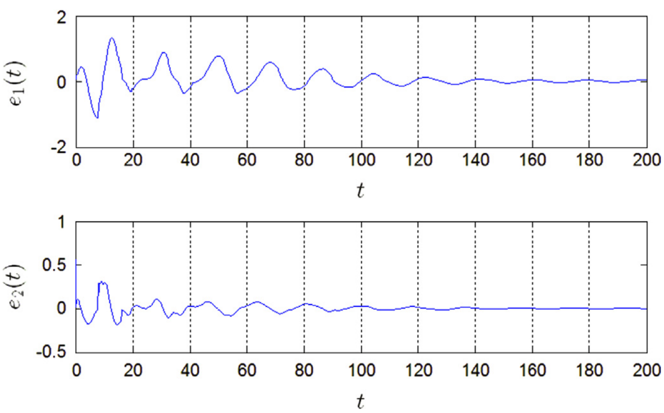

Fig. 13. The evolution of $e(t)$ with switching topologies.

$$L_2 = \begin{bmatrix} 0.5 & 0 & -0.5 & 0 & 0 \\ -0.6 & 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & -0.8 \\ 0 & -0.8 & -0.6 & 1.4 & 0 \\ 0 & 0 & 0 & -0.3 & 0.3 \end{bmatrix}. \qquad (45)$$

And the adjacency matrix of information exchanges between G_1 and G_2 is the same as that in Example 1.

Moreover, systems (2) and (4) starts at (a), and switches every $T=20$ s between (a) and (b). According to Eqs. (2), (4), (28) and (29), the evolutions of q_i and $\dot{q}_i$ for these ten agents are shown in Figs. 9 and 10 while the evolutions of x_i and v_i are shown in Figs. 11 and 12, respectively. Therefore, the protocols (28) and (29) solve the group consensus problem with switching topologies. Furthermore, the evolution of $e(t)$ is shown in Fig. 13, which implies that the protocols (28) and (29) can also solve the time-delay group consensus problem. It should be noted that the dotted line represents the switching time.

5. Conclusion

This paper discussed the group consensus problem of the heterogeneous agents which are governed by either the Euler–Lagrange system or the double-integrator system. In this study, the parameters of the Euler–Lagrange system are uncertain. In order to achieve group consensus, a novel group consensus protocol and a time-varying estimator of the uncertain parameters have been proposed. By combining algebraic graph theory with the Barbalat lemma, several effective sufficient conditions were obtained. Compared with the existent works in the literature, we proposed the definition of the time-delay group consensus, and proved that the time-delay group consensus can be achieved suppose that the inner coupling matrices are equal in the different sub-networks. Besides, the paper also considered switching topologies between homogeneous agents. With the help of the Barbalat-like lemma, it was proved that the group consensus problem with switching topologies can be solved. Finally, two numerical simulations were presented to illustrate the theoretical results. In the future, time-delay needs to be considered in the information exchanges between homogeneous agents and the corresponding group consensus protocols need to be designed.

Acknowledgment

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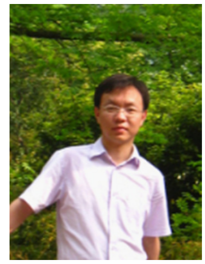

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