Role of lurkers in threshold-driven information spreading dynamics

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(Received 23 November 2020; revised 25 May 2021; accepted 9 September 2021; published 21 September 2021)

The threshold model as a classical paradigm for studying information spreading processes has been well studied. The main focuses are on how the underlying social network structure or the size of initial seeds can affect the cascading dynamics. However, the influence of node characteristics has been largely ignored. Here, inspired by empirical observations, we extend the threshold model by taking into account lurking nodes, who rarely interact with their neighbors. In particular, we consider two different scenarios: (i) Lurkers are absolutely silent and never interact with others and (ii) lurkers intermittently interact with their neighborhood with an activity rate p. In the first case, we demonstrate that lurkers may reduce the effective average degree of the underlying network, playing a dual role in spreading dynamics. In the latter case, we find that the stochastic dynamic behavior of lurkers could significantly promote the spread of information. Concretely, slightly raising the activity rate p of lurkers may result in a remarkable increase in the final cascade size. Further increasing p could make nodes become more stable on average, while it is still easy to observe global cascades due to the fluctuations of the effective degree of nodes.

DOI: 10.1103/PhysRevE.104.034308

I. INTRODUCTION

Modern social media such as Twitter, Facebook, and WeChat have profoundly changed the way people communicate, which makes it more difficult to characterize information spreading processes. To understand the emerging patterns and the underlying mechanisms of information diffusion in social networks, many efforts have been made by researchers, from empirical studies to theoretical modeling [1–6]. In particular, the model-based approach has attracted plenty of interest. For example, the Daley-Kendall model [7–10] and epidemic models [11–14] have been widely employed in modeling information cascades, with the potential to explain a variety of simple contagion (implying that exposure to a single source may cause the state conversion of an individual) phenomena in reality. In contrast, the spread of controversial or risky information often requires social reinforcement from multiple sources, known as complex contagion [15,16]. In this context, the behavior change of an individual is usually determined by comparing the agent's threshold with all neighboring states [17–21]. Based on this assumption, the threshold model was first proposed by Schelling [18].

In the threshold model, it is assumed that each individual in a system can be in one of two states: Susceptible or infected. Susceptible nodes will convert to an infected state if at least a certain fraction of its neighbors are infected. The physical properties of the threshold model were thoroughly studied by Watts [19], who showed that there is a cascade window in the parameter space consisting of the average threshold ϕ and the average degree z of the underlying network, in which global cascades can be observed (a global cascade is defined as a sufficiently large cascade that occupies a finite fraction of nodes if the network is infinitely large).

Watts's seminal work has triggered a great deal of attention recently, especially in the field of network science. The majority of the related studies focus on the effect of network structures or initial size of seeds on cascading dynamics [22–28], as well as the temporal behavior of spreading processes [29,30]. Despite the great progress, there is still a gap between the Watts model (as well as its variations) and the real spreading phenomena. For instance, the current models assume that a node could perceive all of its neighbors simultaneously and is thus aware of the state of each neighbor at all times. Nevertheless, some empirical observations show that in real social networks, there are plenty of lurkers who are seldom exposed to their friends [see Fig. 1(a)]; they may receive information from their neighbors, but rarely send messages to them. How this behavior may influence the information spreading dynamics, however, has not been fully explored.

In this paper we extend the threshold model by introducing lurking nodes and discuss their effect on the cascading dynamics. In particular, we assume that lurkers are generally in a silent state, but may become active and contact their neighbors with a certain rate p. For $p = 0$, we present analytical and numerical results and show that lurkers play a dual role in the spreading process. For $p > 0$, we find that the cascade dynamics is totally different compared to the case of $p = 0$.

This paper is organized as follows. In Sec. II we present the extended threshold model by considering lurkers. In Sec. III we analyze the special situation that lurkers are absolutely silent (i.e., $p = 0$). In Sec. IV we consider the general case that lurkers may become active with rate $p > 0$. We summarize in Sec. V.

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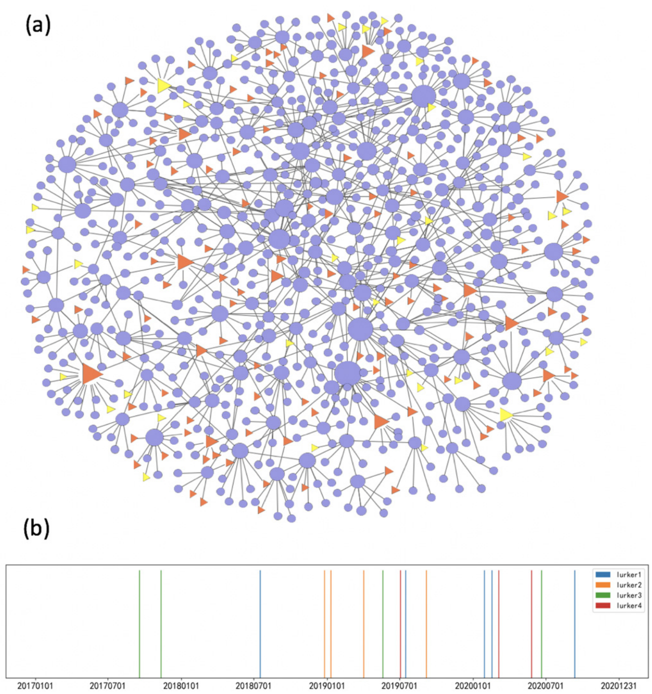

FIG. 1. (a) Sample of the Xianyu (an online social medium in China that serves secondhand sales) social network. This subnetwork has 863 nodes and 1015 links, with nodes representing users and links representing friend relationships. The triangles denote the set of users who had at least one login record in the last three months before the reference date, 14 September 2020 (hinting that the user accounts were not abandoned), and posted a few messages ($\leqslant 6$). The yellow triangles indicate that the users never posted any message. These nodes might be lurkers, accounting for 16.3% of the whole subnetwork. (b) Activities (posting messages) of four randomly chosen lurkers, with a time period starting from January 2017 to September 2020. The average interevent time for a lurker is generally quite long, corresponding to many months.

II. MODEL

The model consists of N individuals forming an Erdős-Rényi (ER) random network with average degree z. Nodes in the system have two possible states: Susceptible (denoted by 0) and infected (denoted by 1). We assume that all nodes are susceptible initially except one randomly chosen node which is in the infected state. In the traditional threshold model, each susceptible node will change its state from 0 to 1 if at least a fraction ϕ of its neighbors are infected, and once a node becomes infected, it will stay in this state until the end of the dynamics. The key point here is that each node could be perceived by its neighbors such that the state of a node is clearly known by its neighbors.

Real social networks, however, are filled with lurkers who rarely interact with others [31–33]. These nodes are silent in general, but may occasionally engage with their neighborhood [as shown in Fig. 1(b)]. To consider this behavior pattern in our model, we randomly select a fraction f of nodes in the network as lurkers, which by default are in a silent state, meaning that they cannot be perceived by their neighbors. In some sense, the lurkers in the silent state are equivalent to being removed from the network, but not exactly, since they still partly participate in the spreading process (for example, they may receive information from others or could be selected as the initial seed). At each time step t, we assume that every lurker in the silent state may become active and activate the links connecting to their neighbors with probability p (usually, $p \ll 1$). At the next time step $t + 1$, all active lurkers become silent again. Define the effective degree of a node as the number of neighbors that can be perceived (called exposed neighbors), which is time dependent due to the activity of lurkers. At time t, a node i with effective degree $k_i^{\text{eff}}(t)$ may become infected if

$$\frac{m_i}{k_i^{\text{eff}}(t)} \geqslant \phi_i, \tag{1}$$

where ϕ_i is the threshold of node i and m_i is the number of infected nodes in the exposed neighborhood. For the sake of simplicity, we assume that all nodes have the identical threshold ϕ in this paper; the qualitative results will not be affected for a heterogeneous distribution of ϕ_i. On average, we have $\langle k_i^{\text{eff}}(t)\rangle = k_i - fk_i + fpk_i$, where k_i is the total degree of node i, including both lurkers and nonlurkers. Obviously, when $f = 0$ or $p = 1$, our model is equivalent to the traditional Watts model. It is worth noting that the lurkers in the silent state are excluded in both the numerator and denominator of the threshold condition (1), i.e., they are not counted when their neighbors consider the decision to change state, which is different from the blocked nodes studied in [29]. This makes sense when people need to make a careful decision; the uncertainty information (like the state of lurkers in silent state) would probably not be taken into account.

III. LURKERS WITH ACTIVITY RATE $p = 0$

We first consider the simple case that lurkers are always silent, i.e., the activity rate of lurkers $p = 0$. Recall that in the original Watts model, there is a phase boundary in the (ϕ, z) space that encompasses a region in which global cascades can occur. Taking the ER network as an example, Fig. 2(a) shows the frequency F_g of global cascades as a function of node threshold ϕ and average degree z (in simulations, if at least 10% nodes in the system become infected, it is determined as a global cascade). The phase diagram can be understood intuitively: Below the lower boundary ($z < 1$), the network is under percolation (i.e., fragmented), so information cannot spread out. For $z \gg 1$, it is hard to satisfy the threshold condition, which results in a situation in which only a few nodes can flip their state. Hence, the frequency of global cascades (as well as the average cascade size) displays a nonmonotonic change as z increases.

By introducing lurkers (with $p = 0$), the phase diagram changes dramatically [as shown in Fig. 2(b)]. Note that the lurkers are removed from the network in some sense. Consequently, the network with average degree z could be replaced by a network with average degree $(1 - f)z$. This may cause some complicated (but not surprising) results. On the one hand, we see that even for large values of z (compared to the original Watts model), it is still possible to observe global cascades, indicating that lurkers can facilitate the spreading of information. On the other hand, many lurkers may dilute the network, requiring more links to form a giant connected

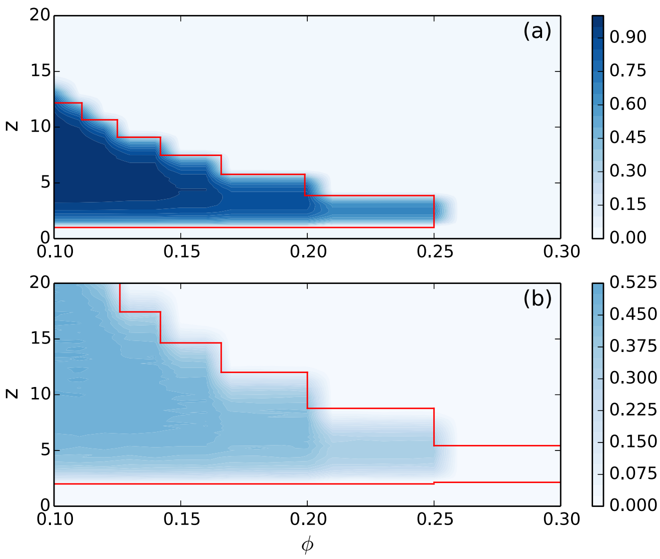

FIG. 2. Frequency F_g of global cascades as a function of node threshold ϕ and average degree z for different values of f: (a) $f = 0$, corresponding to the original threshold model, and (b) $f = 0.5$. The red lines show the boundary of the global cascade regime according to Eq. (7). Simulations correspond to an ER network with size $N = 5000$ and are averaged over 10^4 realizations.

cluster. Hence, the lower boundary of the phase diagram shifts upward (the simulation results show that $z_c \approx 2$), suggesting that lurkers also play an inhibitory role in the spreading dynamics. The dual role of lurkers is mainly caused by the reduction in the effective average degree of the network.

The above arguments can further be confirmed by investigating how the final average cascade size S_∞ changes with f. As shown in Fig. 3, the behavior of $S_\infty(f)$ depends on the position of the paired parameters (ϕ, z) inside the original cascade window [see Fig. 2(a)]. For example, for $\phi = 0.18$ and

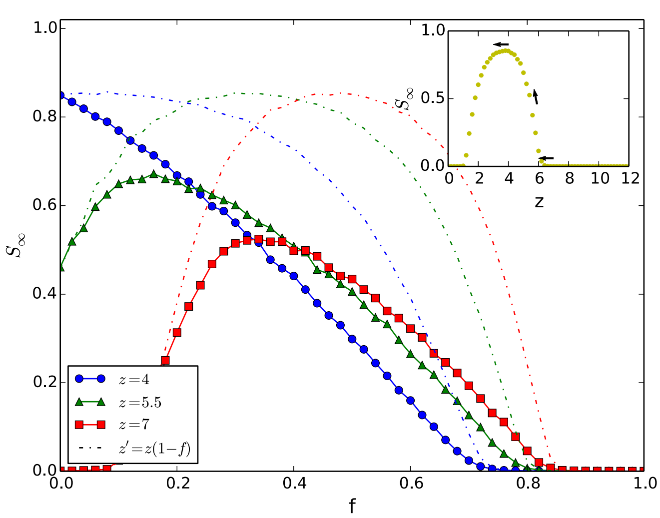

FIG. 3. Final average cascade size S_∞ as a function of f for different values of average degree z, compared with the results on ER networks with average degree $z' = z(1 - f)$ (dashed lines). The inset shows how S_∞ changes with z in the original Watts model. The parameters in the simulations are $N = 5000$ and $\phi = 0.18$. All results are averaged over 10^4 realizations.

$z = 4$ (see the blue circles in Fig. 3), the final average cascade size drops monotonically as f increases until a critical value f_c, at which the average cascade size becomes 0 (implying that no global cascade is possible). In contrast, as z increases, S_∞ exhibits a nonmonotonic change with f: It first grows and then drops, meaning that there is an optimal value of f for the large-scale spread of information. These results can be easily understood if we notice that the increase in f is to some extent equivalent to the decrease in the effective average degree (see the inset in Fig. 3). For comparison, the corresponding results on ER random networks with average degree $z' = (1 - f)z$ are also displayed (without lurkers; see the dashed lines). We see that the two cases are similar but not exactly the same: S_∞ is suppressed in our model since there is a possibility for selecting a lurker as the initial seed, which reduces the chance for global cascades.

In this simple case, our model can be analyzed by the treelike approximation [23,28], which requires that the network structure is locally treelike, i.e., without loops. To proceed, we replace the ER random network by a tree structure. The top level of the tree is a single node (labeled by i) with degree k_i. Each of the direct neighbors of this node (labeled by j) has a degree k_j, with $k_j - 1$ neighbors at the next level, and so on. We label the tree from the bottom with $n = 0$ to the top with $n = \infty$. The probability that a randomly chosen node from level n (excluding the top one) has k neighbors is $\tilde{p}(k) = kp(k)/z$ [28], which is the degree distribution of the neighbors for a node in a graph. Initially, a fraction ρ_0 of nodes are in infected states (in our model, $\rho_0 = 1/N$). A key assumption of this approach is that the nodes will update their states level by level, from bottom to top sequentially, i.e., a node at level n can only be affected by the nodes from the level $n - 1$.

Let us consider a randomly chosen node at level $n + 1$. The probability that it has k neighbors is $\tilde{p}(k)$: One is on level $n + 2$ (called parent) and the left $k - 1$ ones are on level n (called children). Since a fraction ρ_0 of nodes are selected at random as initial seeds, the chosen node is infected with probability ρ_0 and susceptible with probability $1 - \rho_0$. In the first case, the node will keep its state unchanged, while in the latter case, due to peer pressure, it will turn from state 0 to 1 with a certain probability. Considering that each node is a lurker with probability f, there are approximately $\lfloor fk \rfloor$ ($\lfloor \cdots \rfloor$ means to round down a fraction) lurking neighbors on average for the chosen node. Hence, the number of exposed neighbors (nonlurkers) is $\Theta_k = k - \lfloor fk \rfloor$. Define q_n as the probability that a randomly selected node on level n is infected, conditioned on its parent node at level $n + 1$ being susceptible. Thus each of the children of the selected node is exposed and infected with probability $u_n = q_n(1 - f)$. Consequently, the probability that the chosen node has m exposed infected children is $\binom{k-1}{m} u_n^m (1 - u_n)^{k-1-m}$. Finally, we obtain the probability that the chosen node (if it is susceptible) turns state from 0 to 1 due to peer pressure

$$G(q_n) = \sum_{k=1}^{\infty} \frac{kp(k)}{z} \sum_{m=0}^{k-1} \binom{k-1}{m} u_n^m (1 - u_n)^{k-1-m} F\left(\frac{m}{\Theta_k}\right), \tag{2}$$

with

$$F\left(\frac{m}{\Theta_k}\right) = \begin{cases} 1 & \text{for } \frac{m}{\Theta_k} \geqslant \phi \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

denoting the probability that the node satisfies the threshold condition. Considering that the randomly chosen node at level $n+1$ can be either infected (with probability ρ_0) or susceptible (with probability $1-\rho_0$), we finally get the recursion relation

$$q_{n+1} = \rho_0 + (1-\rho_0)G(q_n), \tag{4}$$

with $q_0 = \rho_0$ (notice that nodes in level 0 cannot be affected by other nodes in the system).

Note that the probability that the single node at the top level has degree k is $p(k)$; thus the probability that the top node is infected at the final state is given by

$$\rho = \rho_0 + (1-\rho_0)\sum_{k=1}^{\infty} p(k)\sum_{m=0}^{k}\binom{k}{m}u_\infty^m(1-u_\infty)^{k-m}F\left(\frac{m}{\Theta_k}\right), \tag{5}$$

where $u_\infty = q_\infty(1-f)$. A global cascade occurs when the value of ρ is large (much larger than ρ_0), which means q_n must grow as n increases, at least initially. Linearizing Eq. (4) near $q=0$, we obtain the condition for global cascades

$$(1-f)\sum_{k=1}^{\infty}\frac{k(k-1)}{z}p(k)F\left(\frac{1}{\Theta_k}\right) \geqslant 1, \tag{6}$$

where we have considered the case that the network is infinitely large and $\rho_0 \to 0$. Inserting the expression $p(k) = e^{-z}z^k/k!$, we get

$$(1-f)e^{-z}\sum_{k=2}^{\infty}\frac{z^k}{(k-2)!}F\left(\frac{1}{\Theta_k}\right) - z \geqslant 0. \tag{7}$$

This equation can be solved numerically. Note that there are three parameters f, ϕ, and z in the equation; we may, for example, vary two parameters f and ϕ and solve for the third one z. The results are shown in Fig. 2 (red lines).

IV. LURKERS WITH ACTIVITY RATE $p > 0$

In this section we consider the general case that $p > 0$, in which lurkers exhibit stochastic dynamic behavior. Figure 4 shows the evolution of the average cascade size S for different values of $p \ll 1$. Given $z = 7$, $f = 0.3$, and $\phi = 0.18$, we see that S grows rapidly to a steady-state value ($S_\infty \approx 0.5$) for $p = 0$, while raising p, even by a little bit, leads to a very large value of S_∞, indicating that the spreading dynamics for $p > 0$ may be essentially different from that for $p = 0$ (see also the blue curve in Fig. 6, where there is a jump at $p = 0$). Moreover, it seems that for different $p > 0$, there is the same asymptotic state, while the evolving process towards it depends on the specific value of the parameter: Smaller p corresponds to a longer time to reach the steady state. Note that changing p may affect the effective average degree [$z_{eff} = z(1-f+fp)$] of the network, which further influences the spreading dynamics. However, for extremely small values of p ($p \sim 0$), the variation in the effective average degree is negligible. Considering this, the striking difference between the cases $p = 0$ and $p \sim 0$ is intriguing.

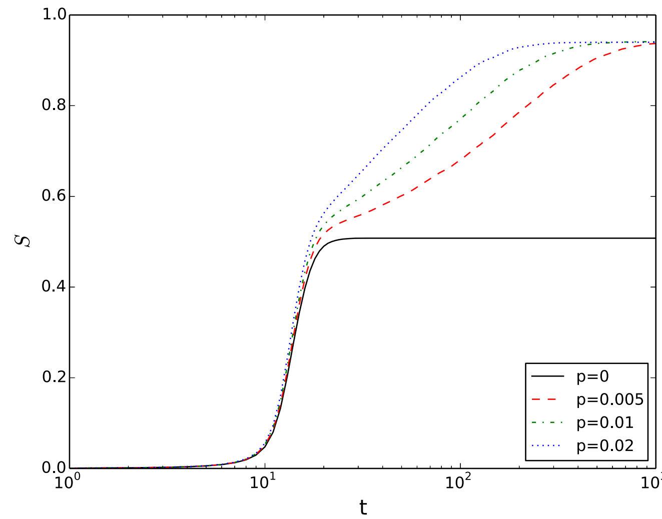

FIG. 4. Average cascade size S_∞ as a function of time t for different values of activity rate p. The parameters in the simulations are $N = 5000$, $f = 0.3$, $\phi = 0.18$, and $z = 7$. The results are averaged over 10^4 realizations.

In fact, the behavior of infected lurkers plays a key role here. For $p = 0$, the lurkers completely block the flow of information to other nodes, while as long as $p > 0$ (no matter how small it is), there is a chance for information to pass along provided the lurker is infected. In this scenario (i.e., $p \sim 0$), the activity rate p of lurkers controls the flow rate of information from the infected lurkers to their neighbors, resulting in various evolution processes as observed in Fig. 4. In contrast, the number of contacts of an infected lurker is almost unchanged for $p \sim 0$, hinting that the infection range would not be affected (thus having the same asymptotic state). On the other hand, the infected lurker could also facilitate its

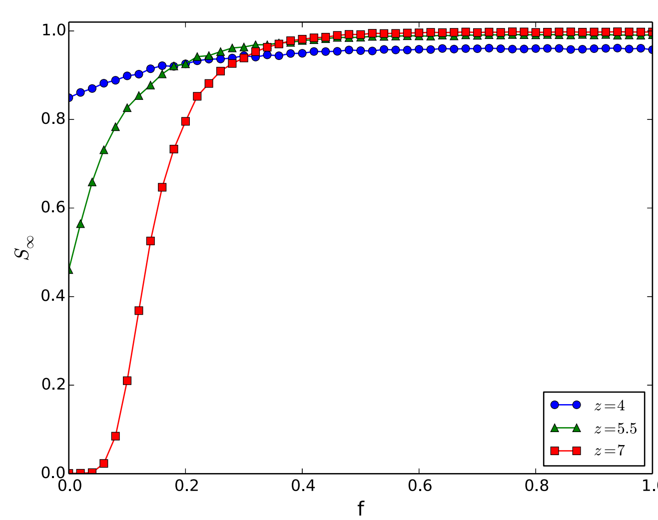

FIG. 5. Final average global cascade size S_∞ as a function of f for different values of z. The parameters in the simulations are $N = 5000$, $\phi = 0.18$, and $p = 0.01$. The results are averaged over 10^4 realizations.

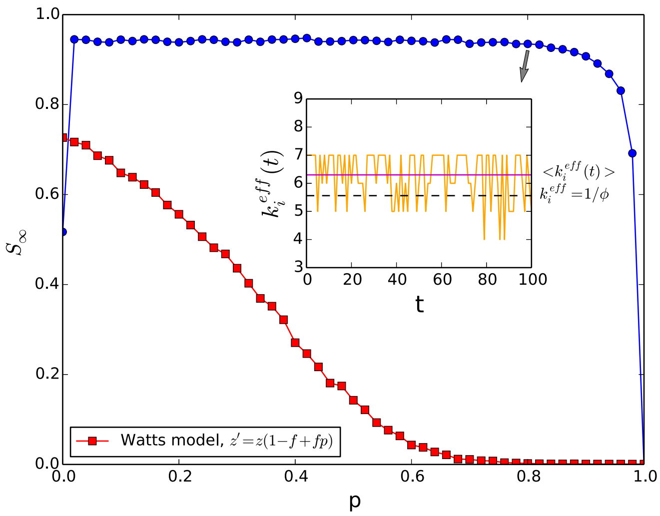

FIG. 6. Final average global cascade size S_∞ as a function of p. As a comparison, the result of the original Watts model on ER networks with average degree $z' = z(1 - f + fp)$ is also presented (red squares). The inset shows the evolution of the effective degree of a randomly chosen node in the network, in which the pink solid line corresponds to the average value and the dashed line corresponds the border $1/\phi$, below which the node is vulnerable. The parameters in the simulations are $N = 5000$, $\phi = 0.18$, $z = 7$, and $f = 0.3$.

neighbors to change state: It contributes in both the numerator and denominator of Eq. (1), making nodes fulfill the threshold condition more easily. Taken together, the behavior of lurkers that occasionally engage with others may significantly promote the spread of information.

Figure 5 further confirms this point, where we show how the final average cascade size S_∞ varies with f for $p = 0.01$. Different from the results as observed in Fig. 3, S_∞ grows monotonically as the fraction of lurkers increases and remains at a high level for large values of f. Note that as f increases, the subnetwork consisting of nonlurkers would gradually fall apart, which may hinder the spreading process in the case of $p = 0$. Nevertheless, for $p > 0$, even though there are plenty of lurkers such that the subnetwork of nonlurkers is separated into many small pieces, information could still spread from one piece to another, percolating through the whole network due to the occasional interactions between lurkers and their neighbors.

Finally, we investigate how the spreading dynamics would be affected by further increasing activity rate p. In this case, the variation in the effective average degree [$z_{eff} = z(1 - f + fp)$] cannot be ignored, indicating that the final cascade size may be noticeably influenced by increasing p. However, we find that in our model, S_∞ is almost unchanged for $0 < p < 1$ (see the blue curve in Fig. 6), which remains at the same high value (approximately equal to 0.94). As a comparison, we also show the result of the original Watts model (i.e., without lurkers) on ER networks with average degree $z' = z_{eff} = z(1 - f + fp)$, where we set z and f and treat z' as a function of p. In simulations, we set the parameters $z = 7$, $\phi = 0.18$, and $f = 0.3$. As expected, the increase in p (or z') results in the decrease in S_∞, accompanying a phase transition (see the red curve in Fig. 6 and refer to Fig. 2). The striking difference in the spreading phenomena could be explained by the dynamical behavior of lurkers. Consider a node i with effective degree $k_i^{\text{eff}}(t)$, which would fluctuate around an average value $\langle k_i^{\text{eff}}(t)\rangle = k_i(1 - f + fp)$ (see the inset of Fig. 6). For large values of p, $\langle k_i^{\text{eff}}(t)\rangle$ may be large enough so that node i is very stable on average. However, due to the fluctuations, $k_i^{\text{eff}}(t)$ may sometimes go across the border $k_i^{\text{eff}} = 1/\phi$, below which the node is vulnerable and could be easily infected (requiring only one infected neighbor to switch state). From this point of view, many stable (on average) nodes in the network are actually vulnerable over a long time span, hinting at the possibility of global cascades for large p.

V. CONCLUSION

Based on the observation that real social networks are inundated with nodes who rarely interact with others, we extended the traditional threshold model by introducing lurkers, in order to understand how these nodes may affect the information spreading dynamics. In particular, we assumed that lurkers could not be perceived by their neighbors in general but, at each time step, would become active and contact their neighbors with a rate p. For $p = 0$, we have shown that lurkers are somehow similar to removed nodes, which may reduce the effective average degree of the underlying network. As a consequence, on the one hand, they make other nodes fulfill the threshold condition easily, playing a positive role in the spreading process. On the other hand, many lurkers may disconnect the network, inhibiting the diffusion of information. Slightly increasing p, however, would totally change the cascade dynamics. In this case (i.e., $p \sim 0$), the effective average degree is almost unchanged (given the fixed number of lurkers), while the average cascade size of information grows remarkably, owing to the dynamic behavior of the infected lurkers. Further increasing p would notably raise the effective average degree, whereas we may still observe global cascades due to the fluctuations of node degree. Our model emphasizes the importance of node characteristics in the information cascade processes in social networks, which may help us better understand the real spreading phenomena.

ACKNOWLEDGMENTS

We acknowledge the anonymous referees for their valuable comments. This work was partially supported by the NNSF of China under Grants No. 11605154 and No. 61973273 and by the Zhejiang Provincial Natural Science Foundation of China under Grants No. LY21F030017 and No. LR19F030001. This research was also supported by the Fundamental Research Funds for the Central Universities (Grant No. 21621007).

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